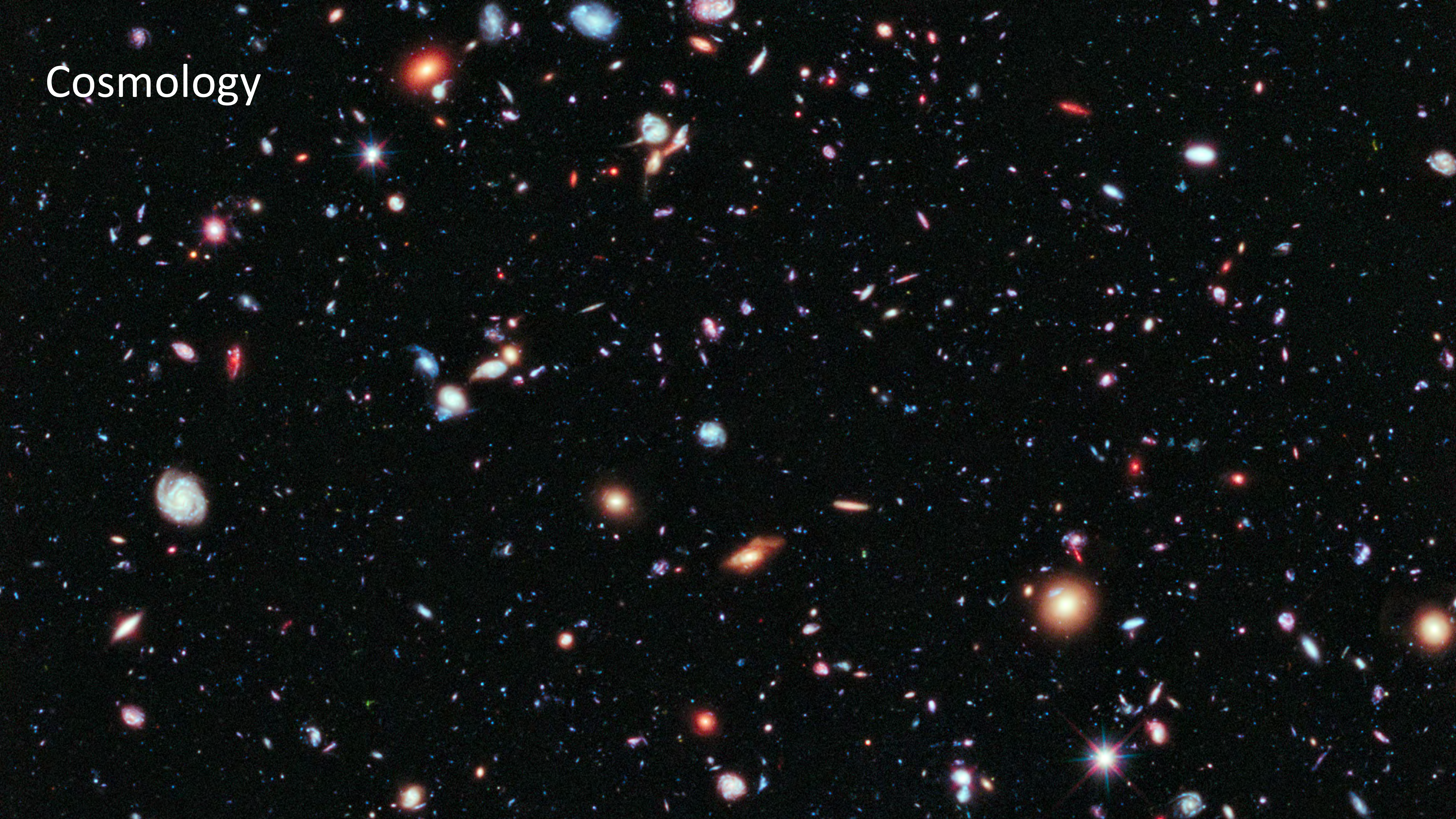
Cosmology

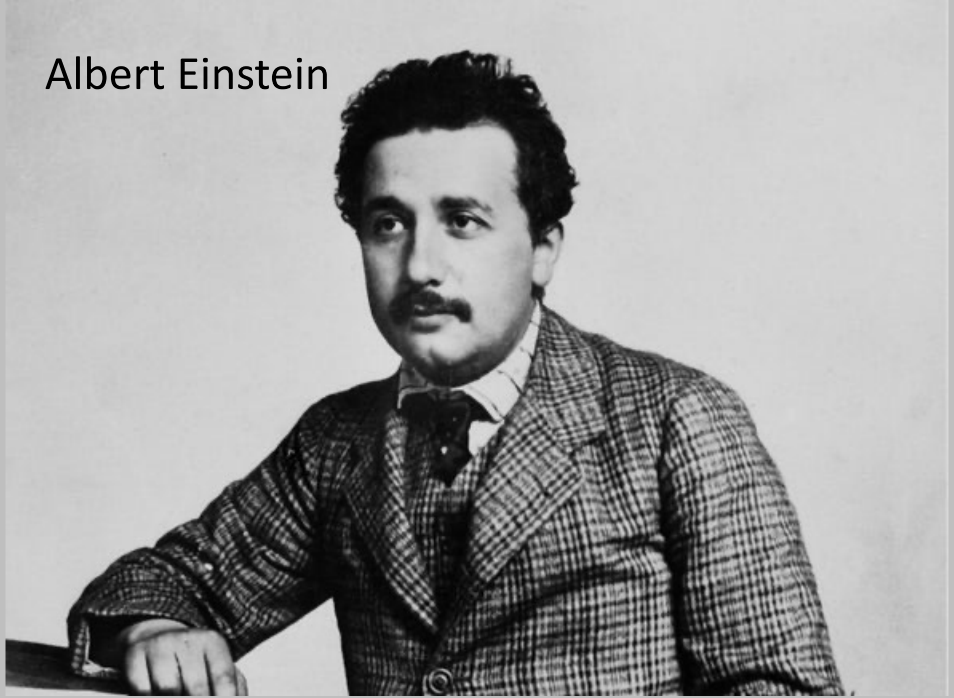


Cosmology

The beginnings of modern cosmology

1915: Einstein works out the model of General Relativity, describing gravitation and its coupling with space and time.

Willem de Sitter uses GR to work out a model that has the Universe expanding in time. Einstein rejects that concept, adding an ad-hoc term to his equations called the cosmological constant which keeps the universe from expanding.



The beginnings of modern cosmology

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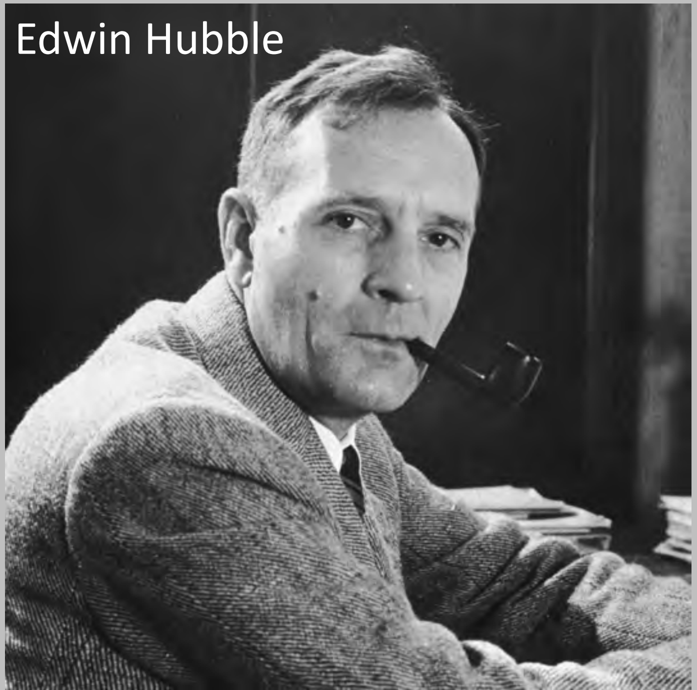
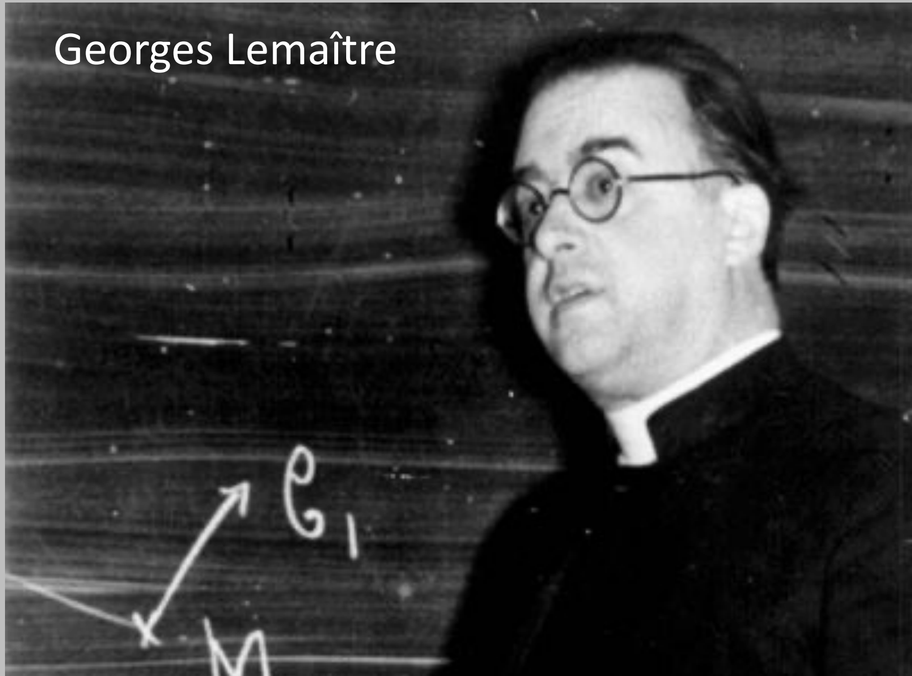
1920s: Alexander **Friedman** and Georges **Lemaître** also put forth models of an expanding universe.

1929: Edwin **Hubble** discovers the expansion of the Universe.

Einstein gives up on the cosmological constant, calling it his ”greatest blunder.”

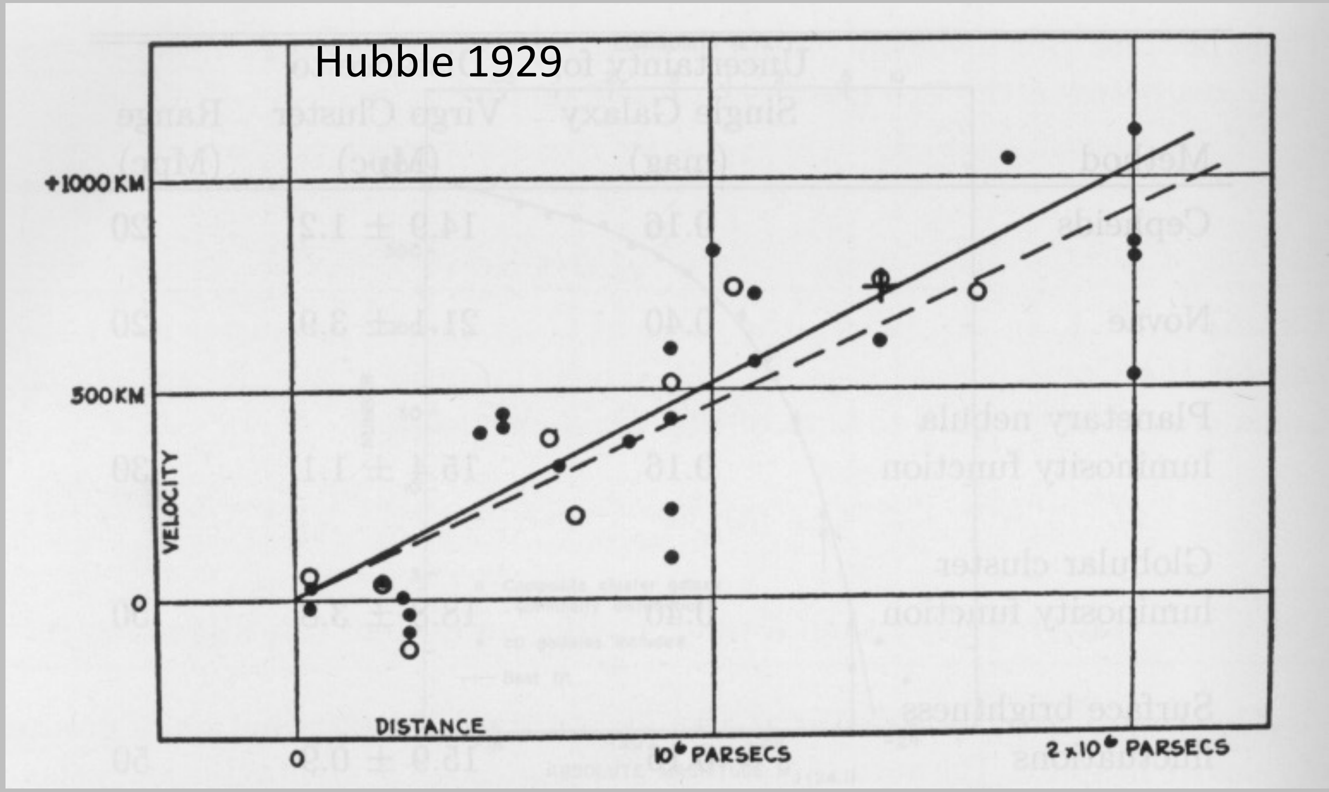




The Expanding Universe

Hubble's Law: $v = H_0 d$, where $H_0 \approx 72$ km/s/Mpc

(Note: Hubble got H_0 wrong, by a lot. H_0 is the slope of that line ⇒ which is about 500 km/s/Mpc. Hubble's incorrect distances were the problem.....)



The true meaning of redshift: cosmological expansion

Remember redshift:

$$z = \frac{\lambda_{obs} - \lambda_0}{\lambda_0} = \frac{\Delta\lambda}{\lambda_0}$$

We originally referred to this using the Doppler shift, but ***this is not correct***. The Universe – space itself – is expanding. As light moves through an expanding Universe, its wavelength is stretched: a redshift.

Important Note: on small scales the expansion is weak and gravitational forces can overcome it. The solar system is not expanding, galaxies are not growing bigger, even galaxy clusters do not expand.

You are not (cosmologically) expanding!

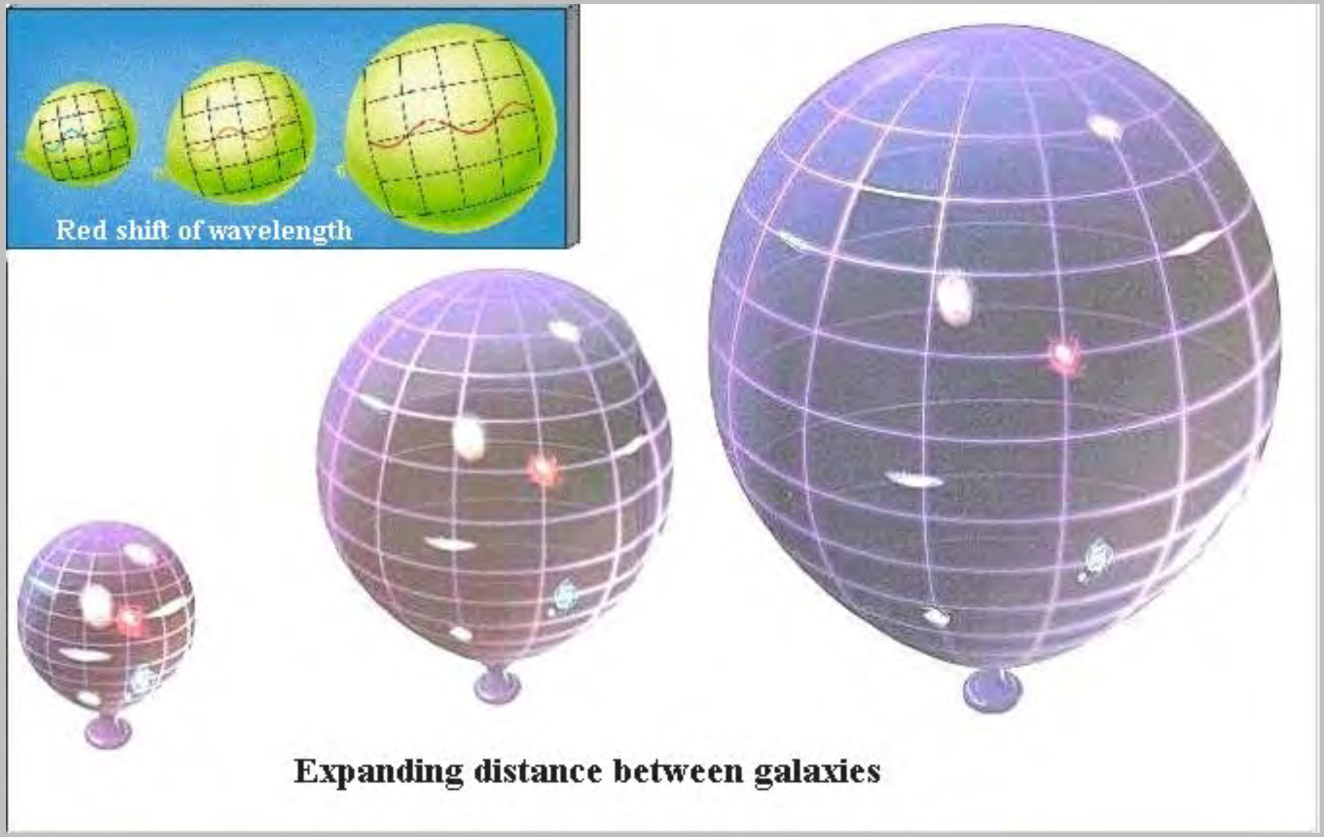
Coordinates and distances in an expanding universe

Do describe coordinates in an expanding universe, we define something called a **co-moving coordinate system** that expands along with the universe. We can then describe a **proper distance** this way

$$d = R(t)r$$

d : proper distance
r : co-moving distance
$R(t)$: dimensionless scale factor describing expansion

We define $t = t_0$ to be now, and $R(t_0) \equiv 1.0$. The scale factor of the universe is exactly one today by definition.



For these reasons, in cosmology we do not talk in terms of distances, we use redshifts.

But the proper distance to a galaxy is not measurable in any conventional sense.

It is not the distance you would have to travel to get to the galaxy.
It is also not the same as the distance the galaxy's light has travelled to get to us.
It is also not the distance you use in the magnitude-distance equation ($m - M = 5 \log d - 5$)
It is also not the distance you use to convert angular size to physical size ($D_{phys} = d \tan \alpha \approx \alpha['']d/206265$)

Redshift and expansion factor

Start with the definition of redshift, writing in terms of the emitting wavelength of light and the observed wavelength of light.

$$z = \frac{\lambda_{obs} - \lambda_{em}}{\lambda_{em}}$$

$$z = \frac{\lambda_{obs}}{\lambda_{em}} - 1$$

Then rearrange terms a bit:

$$1 + z = \frac{\lambda_{obs}}{\lambda_{em}}$$

The stretching of light is due to the expansion of the universe, and is just given by the ratio of the scale factor. If the universe doubled in size, the wavelength was stretched by a factor of two.

$$1 + z = \frac{R_{obs}}{R_{em}}$$

And since we are observing today, and have defined the scale factor of the universe today to be exactly one:

$$1 + z = \frac{1}{R_{em}}$$

So scale factor is measurable!

If we detect light from a galaxy that has a redshift of $z = 3$, we are seeing the galaxy as it was when the scale factor was

$$R_{em} = 1/(1+z)$$
$$= 1/4$$

So we see the galaxy as it was when the universe was a quarter of its current size.

The age of the Universe

If the universe is expanding, then if we run it backwards it must have had a time when $R(t) \to 0$. The beginning!

When was that? Homer Simpson math time.

An object moving at a constant speed covers a distance

$$d = vt$$

or equivalently, to cover a distance d it will take a time

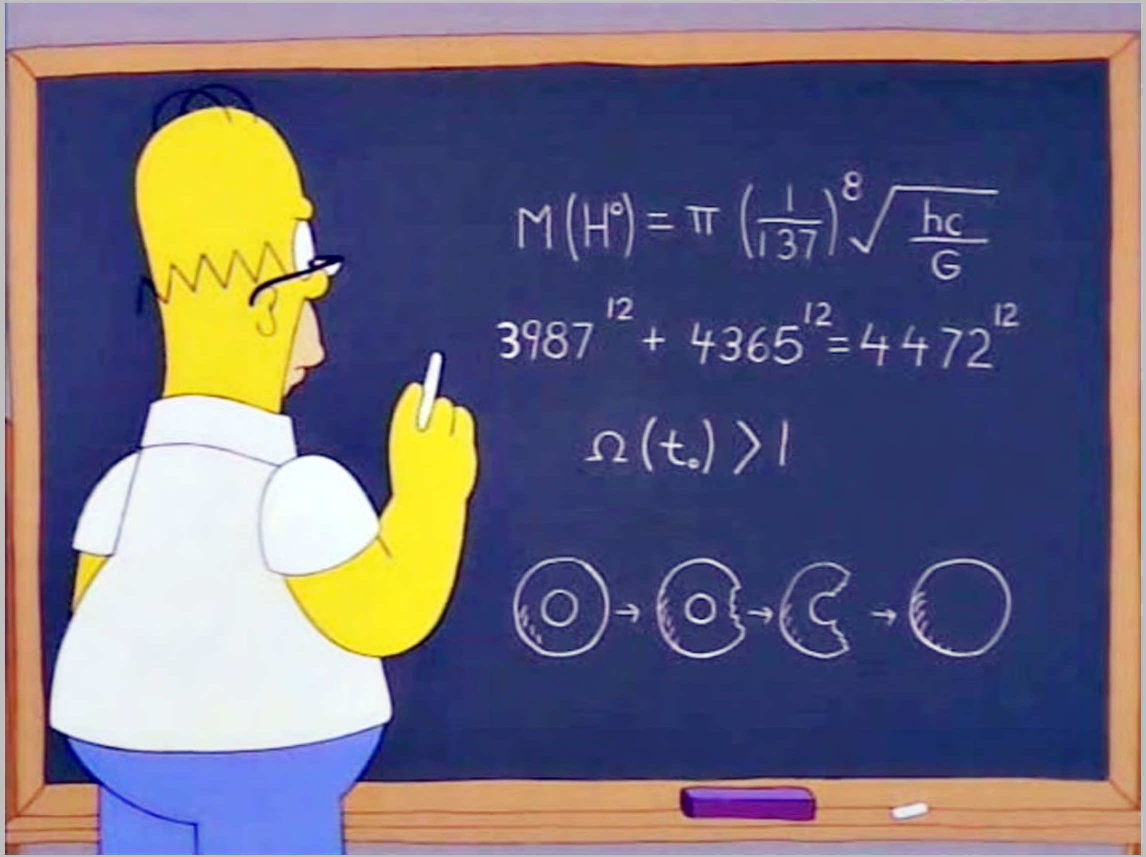
$$t = d/v$$

Now think of a galaxy at a distance d. If the expansion rate has been constant over time, that expansion rate is given by the Hubble constant: $v = H_0 d$.

Then $t_0 = d/v = d/(H_0 d) = 1/H_0$.

Homer says the age of the Universe is $1/H_0$.

What is the big assumption built in?
Constant expansion rate.

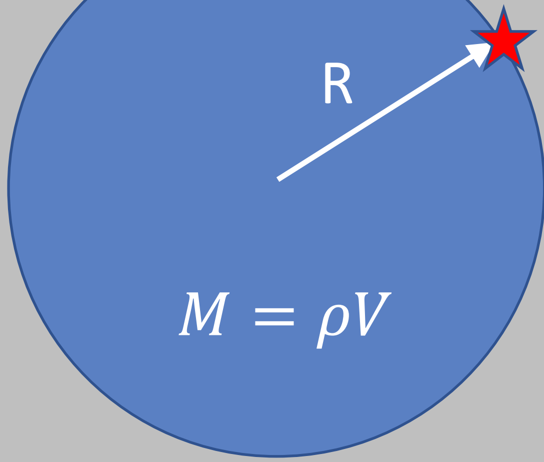


$H_0 = 72$ km/s/Mpc
$H_0 \approx 72$ pc/Myr/Mpc
$= 72\times10^{-6}$ Mpc/Myr/Mpc = 7.2×10^{-5} Myr^{-1}

Then $t_0 = 1/H_0 \approx 1.39\times10^{4}$ Myr = 13.9 billion years!

Solving for R(t): Newtonian Cosmology

Think of a particle on an expanding sphere with size R and total mass M. What is its equation of motion?



Notation:
R: distance from center (changes with time)
$\dot{R}$: 1st derivative w.r.t time (dR/dt, or velocity)
$\ddot{R}$: 2nd derivitive w.r.t time (d^2R/dt^2, or acceleration)

Also:
ρ : density of sphere (changes with time)
ρ_0 : density of sphere today (a fixed value)

$$\ddot{R} = -\frac{GM}{R^2} = -\frac{4\pi}{3}G\rho R$$
⇐ replacing M with $\rho \times V$

$$\rho = \rho_0 R^{-3}$$
⇐ density scales with volume

$$\ddot{R} = -\frac{4\pi}{3}\frac{G\rho_0}{R^2}$$
⇐ so substitute to get this

$$\dot{R}\ddot{R} + \frac{4\pi}{3}\frac{G\rho_0}{R^2}\dot{R} = 0$$
⇐ multiply both sides by $\dot{R}$ to get this

$$\frac{1}{2}\frac{d(\dot{R}^2)}{dt} + \frac{4\pi}{3}\frac{G\rho_0}{R^2}\frac{dR}{dt} = 0$$
⇐ substitute using this relation
$$\frac{d(\dot{R}^2)}{dt} = 2\dot{R}\ddot{R}$$

$$\frac{d}{dt}\left[\dot{R}^2 - \frac{(8\pi G\rho_0/3)}{R}\right] = 0$$
⇐ substitute using this relation
$$\frac{1}{R^2}\frac{dR}{dt} = -\frac{d(1/R)}{dt}$$

$$\dot{R}^2 - \frac{(8\pi G\rho_0/3)}{R} = -k$$
⇐ if $\frac{df}{dt} = 0$, then f is a constant

Solving for R(t): Newtonian Cosmology

so we had

$$\dot{R}^2 - \frac{(8\pi G\rho_0/3)}{R} = -k$$

rewrite this as

$$\dot{R}^2 = \frac{(8\pi G\rho_0/3)}{R} - k$$

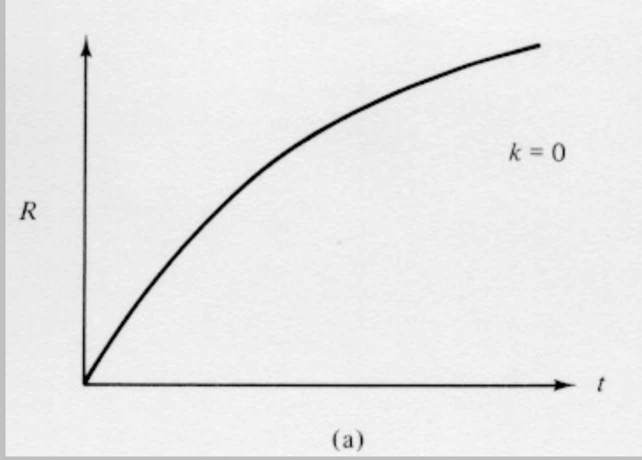
to look at behavior over time.

Now, one final rewrite: replace ρ_0 with ρR^3 and divide everything by R^2 to get:

$$\left(\frac{\dot{R}}{R}\right)^2 - \frac{8}{3}\pi G\rho = -\frac{k}{R^2}$$

Possibilities for k:

$\boldsymbol{k = 0}$: Then $\dot{R}$ is always positive, so the sphere always expands, but at an ever-slowing rate: $\dot{R} \to 0$ as $R \to \infty$.



Solving for R(t): Newtonian Cosmology

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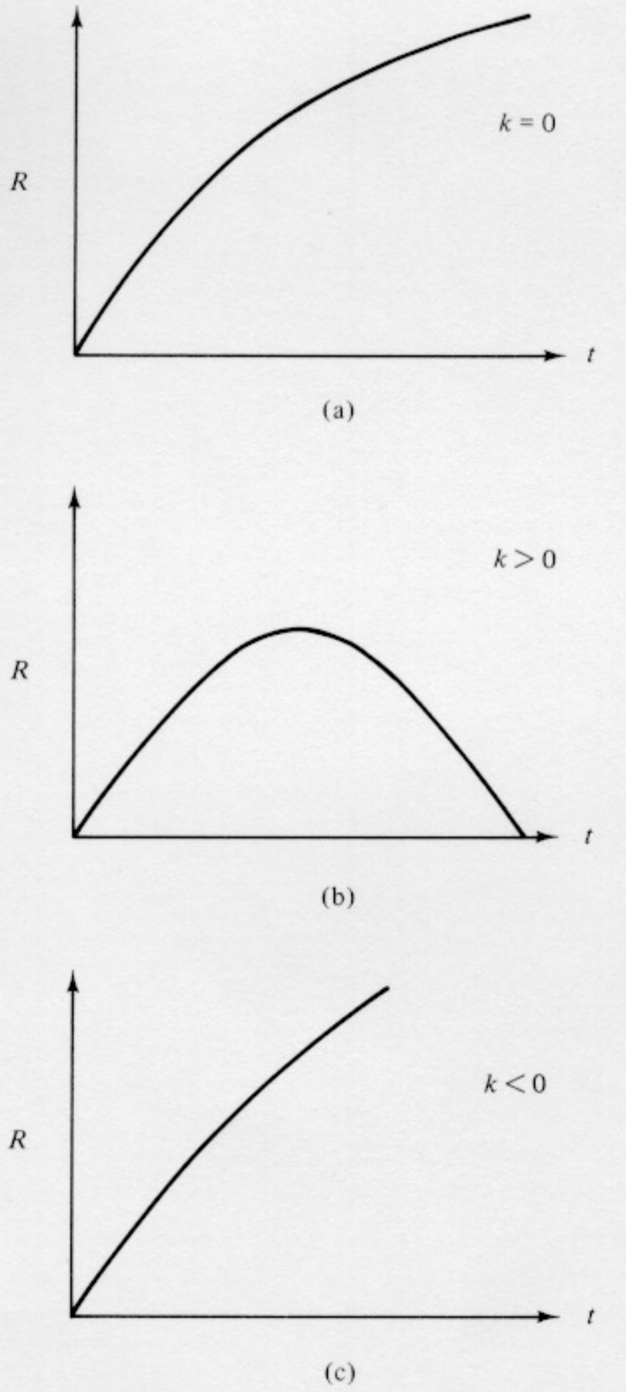
Possibilities for $\boldsymbol{k}$:

$\boldsymbol{k = 0}$: Then $\dot{R}$ is always positive, so the sphere always expands, but at an ever-slowing rate: $\dot{R} \rightarrow 0$ as $R \rightarrow \infty$.

$\boldsymbol{k > 0}$: $\dot{R}$ is initially positive but at some point will reach zero, then gravity wins and the sphere begins to contract.

$\boldsymbol{k < 0}$: $\dot{R}$ is always positive and so the universe is always expanding.



The Friedmann Equation

So under Newtonian dynamics we had

$$\left(\frac{\dot{R}}{R}\right)^2 - \frac{8}{3}\pi G\rho = -\frac{k}{R^2}$$

Surely a proper derivation using General Relativity can't look anything like that, right?

Actually, it can. Solving the Einstein field equations for an isotropic, homogenous universe gives the **dynamics equation**:

$$\frac{\ddot{R}}{R} = -\frac{4\pi}{3}G\rho + \frac{1}{3}\Lambda c^2$$

which solves to the **Friedmann Equation**

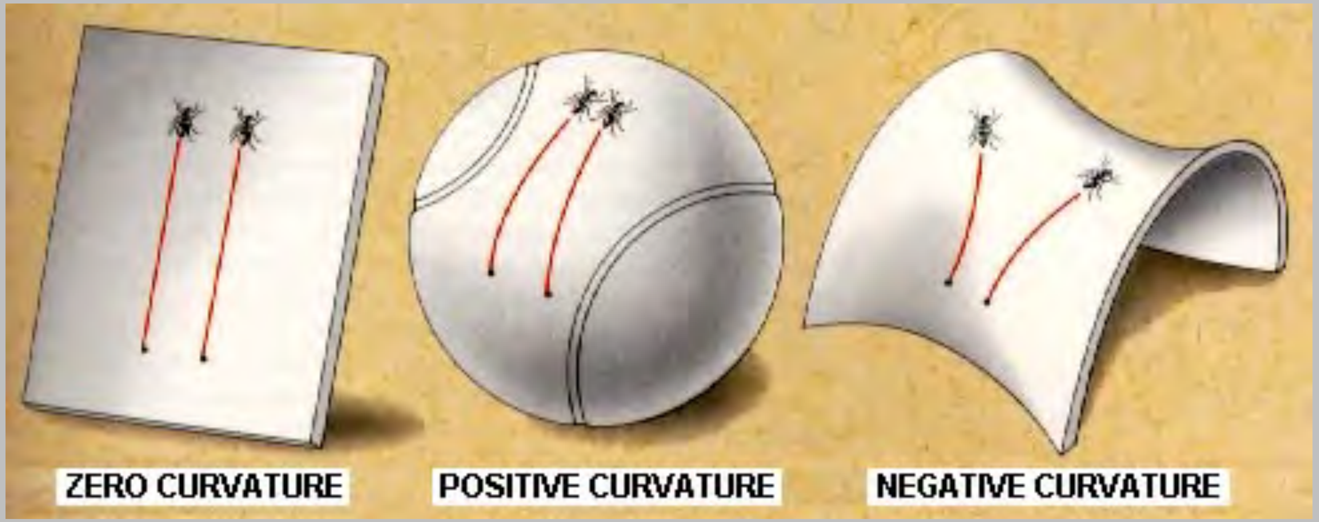
$$\boxed{\left(\frac{\dot{R}}{R}\right)^2 - \frac{8}{3}\pi G\rho - \frac{1}{3}\Lambda c^2 = -\frac{kc^2}{R^2}}$$

Important Notes:

1. The dynamics equation says the without the cosmological constant, the universe can't be static. This is why Einstein introduced the cosmological constant: to get a static universe.

2. Under GR, k is linked to the **curvature of space**.

$k = 0$ $\quad$ $k > 0$ $\quad$ $k < 0$



3. *If there's no cosmological constant* ($\Lambda = 0$), the curvature of space and the expansion history are tightly connected.

$k = 0$	$k > 0$	$k < 0$
universe continually slows	universe eventually recollapses	universe expands forever

$$\left(\frac{\dot{R}}{R}\right)^2 - \frac{8}{3}\pi G\rho - \frac{1}{3}\Lambda c^2 = -\frac{kc^2}{R^2}$$

The (Basic) Cosmological Parameters

The Hubble Parameter ($\boldsymbol{H}$):

$$H \equiv \frac{\dot{R}}{R}$$

H is the normalized rate of expansion, and changes with time as the universe expands.

Its value at the current time (i.e., at $t = t_0$), it is called the Hubble constant, H_0.

Current measures put $H_0 = 72$ km/s/Mpc or so.

$$\left(\frac{\dot{R}}{R}\right)^2 - \frac{8}{3}\pi G\rho - \frac{1}{3}\Lambda c^2 = -\frac{kc^2}{R^2}$$

The (Basic) Cosmological Parameters

The matter density parameter (Ω_m):

Rewrite the Friedmann equation using the Hubble parameter and setting $\Lambda = 0$:

$$H^2 - \frac{8}{3}\pi G\rho = -\frac{kc^2}{R^2}$$

$k = 0$ means the universe is spatially flat, so a “no lambda” universe ($\Lambda = 0$) universe is spatially flat ***if*** it has a critical density ρ_{crit} given by:

$$\rho_{crit} = \frac{3H^2}{8\pi G}$$

We define the matter density parameter as

$$\Omega_m = \frac{\rho}{\rho_{crit}}$$

Current measures put $\Omega_{m,0} \approx 0.30$

The matter density parameter tells you if gravity alone is sufficient to “flatten” the Universe (i.e., if $\Omega_m = 1$).

The data says it is *not*.

$$\left(\frac{\dot{R}}{R}\right)^2 - \frac{8}{3}\pi G\rho - \frac{1}{3}\Lambda c^2 = -\frac{kc^2}{R^2}$$

The (Basic) Cosmological Parameters

The “dark energy” density parameter (Ω_Λ):

Rewrite the Friedmann equation using the Hubble parameter and setting $\rho = 0$:

$$H^2 - \frac{1}{3}\Lambda c^2 = -\frac{kc^2}{R^2}$$

$\mathrm{k} = 0$ means the universe is spatially flat, so a matter-free universe ($\rho = 0$) universe is spatially flat if it a critical value of Λ of

$$\Lambda_{crit} = \frac{3H^2}{c^2}$$

We define the dark energy density parameter as

$$\Omega_\Lambda = \frac{\Lambda}{\Lambda_{crit}}$$

Current measures put $\Omega_{\Lambda,0} \approx 0.7$

The dark energy density parameter tells you if Λ alone is sufficient to “flatten” the Universe (i.e., if $\Omega_\Lambda = 1$).

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$$\left(\frac{\dot{R}}{R}\right)^2 - \frac{8}{3}\pi G\rho - \frac{1}{3}\Lambda c^2 = -\frac{kc^2}{R^2}$$

The (Basic) Cosmological Parameters

“Total Omega” (Ω): $\boldsymbol{\Omega = \Omega_m + \Omega_\Lambda}$

Total Omega tells you if gravity and dark energy combined can make the universe spatially flat (if $\Omega = 1$):

Current estimates say $\Omega_0 = \Omega_{\mathrm{m},0} + \Omega_{\Lambda,0} \approx 0.3 + 0.7 = 1.0$

The universe is spatially flat!

Cosmological Parameters

1. The **Hubble Constant** (normalized expansion rate today):

$$H_0 = \left(\dot{R}/R\right)_{t=t_0} \approx 68 - 72 \text{ km/s/Mpc}$$

More generally, the *Hubble Parameter* (changes with time)

$$H \equiv \dot{R}/R$$

2. The **Matter Density parameter** (normalized mass density):

$$\Omega_m = \frac{\rho}{\rho_{crit}}, \qquad \Omega_{m,0} \approx 0.3$$

3. The **Dark Energy Density parameter** (normalized dark energy density):

$$\Omega_\Lambda = \frac{\Lambda}{\Lambda_{crit}}, \qquad \Omega_{\Lambda,0} \approx 0.7$$

The Dynamics Equation

$$\frac{\ddot{R}}{R} = -\frac{4\pi}{3}G\rho + \frac{1}{3}\Lambda c^2$$

The Friedman Equation

$$\left(\frac{\dot{R}}{R}\right)^2 - \frac{8}{3}\pi G\rho - \frac{1}{3}\Lambda c^2 = -\frac{kc^2}{R^2}$$

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The R(t) plot: Understanding the parameters graphically and intuitively

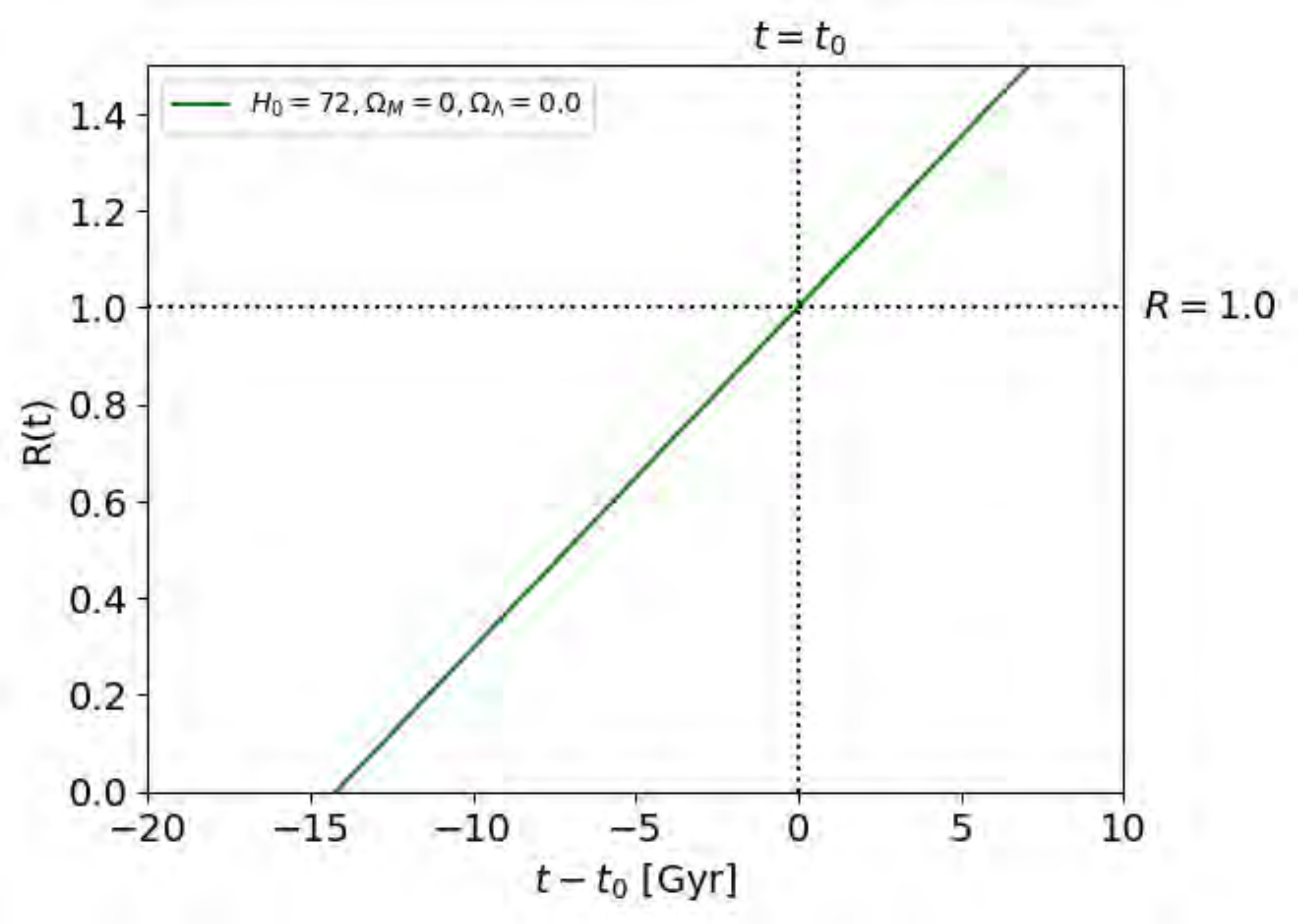
Imagine a Universe that is expanding at a constant rate (Homer Simpson Universe).

In this Universe, R(t) is a straight line, and Homer worked out an age of 13.9 billion years for $H_0 = 72$ km/s/Mpc.

The Hubble Parameter is given by

$$H \equiv \frac{\dot{R}}{R}$$

so it's the slope of the R(t) line divided by the scale factor itself (R).



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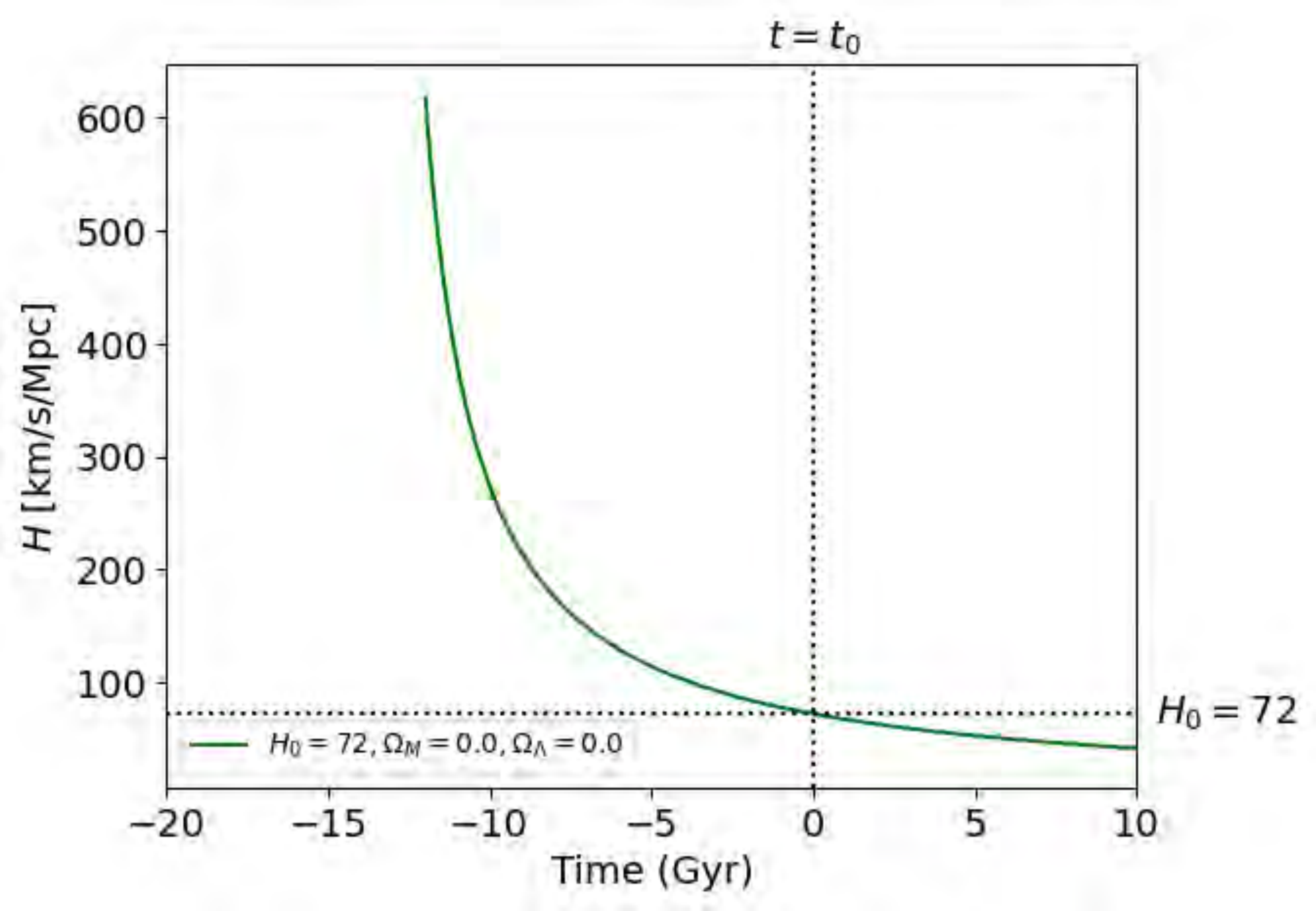

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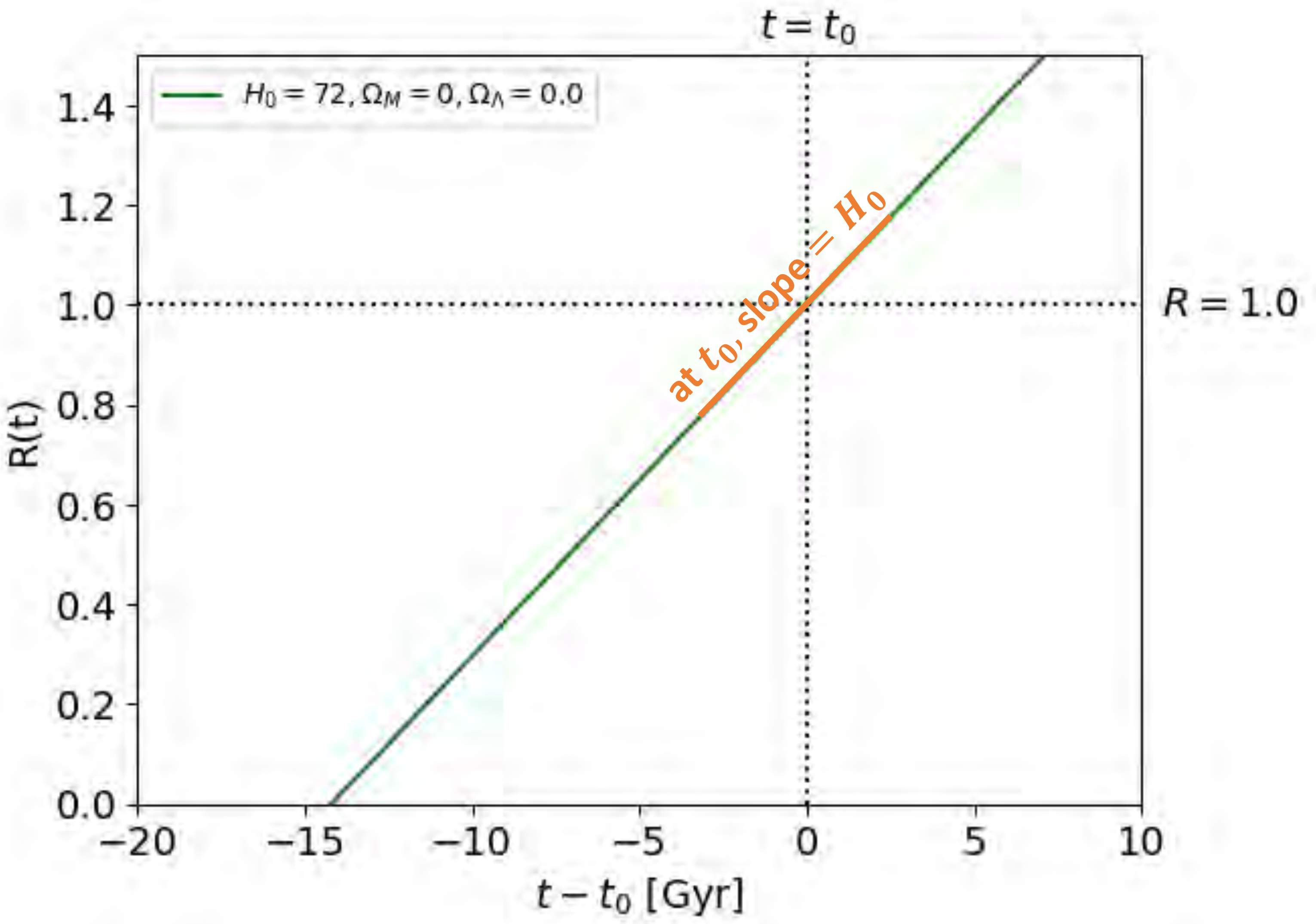
In this Universe, R(t) is a straight line, and Homer worked out an age of 13.9 billion years for $H_0 = 72$ km/s/Mpc..

The Hubble Constant is the Hubble Parameter **today** (at $t = t_0$).

So it is the slope of the line today, divided by $R_0 = 1$.

$$H_0 \equiv \frac{\dot{R}(t_0)}{R_0} = \dot{R}(t_0)$$

So the Hubble Constant is *essentially* just the slope of the line today.



The R(t) plot: Understanding the parameters graphically and intuitively

Imagine a Universe that is expanding at a constant rate (Homer Simpson Universe).

Changing the Hubble Constant means changing the slope at $t = t_0$.

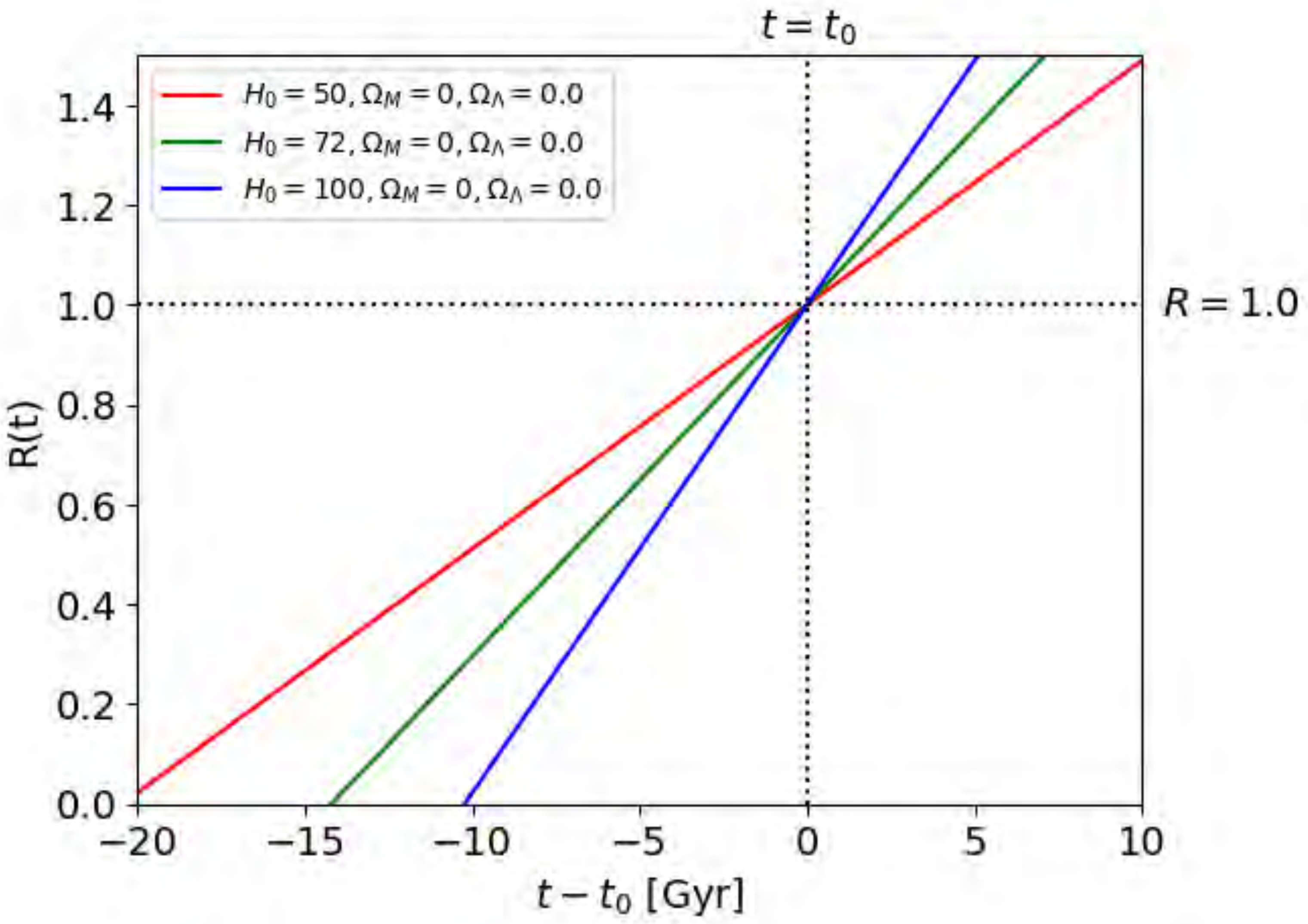
Remember the boundary conditions! No matter how you change the parameters of the Universe, R(t) has to satisfy the following conditions:

- $R(t_0) = 1$
- $H(t_0) = \left(\frac{\dot{R}(t_0)}{R_0}\right) = \dot{R}(t_0) = H_0$

This changes t_0, the age of the Universe! This is why, for a contant expansion universe,

$$t_0 = 1/H_0$$

A higher Hubble constant results in a younger Universe, and vice-versa.

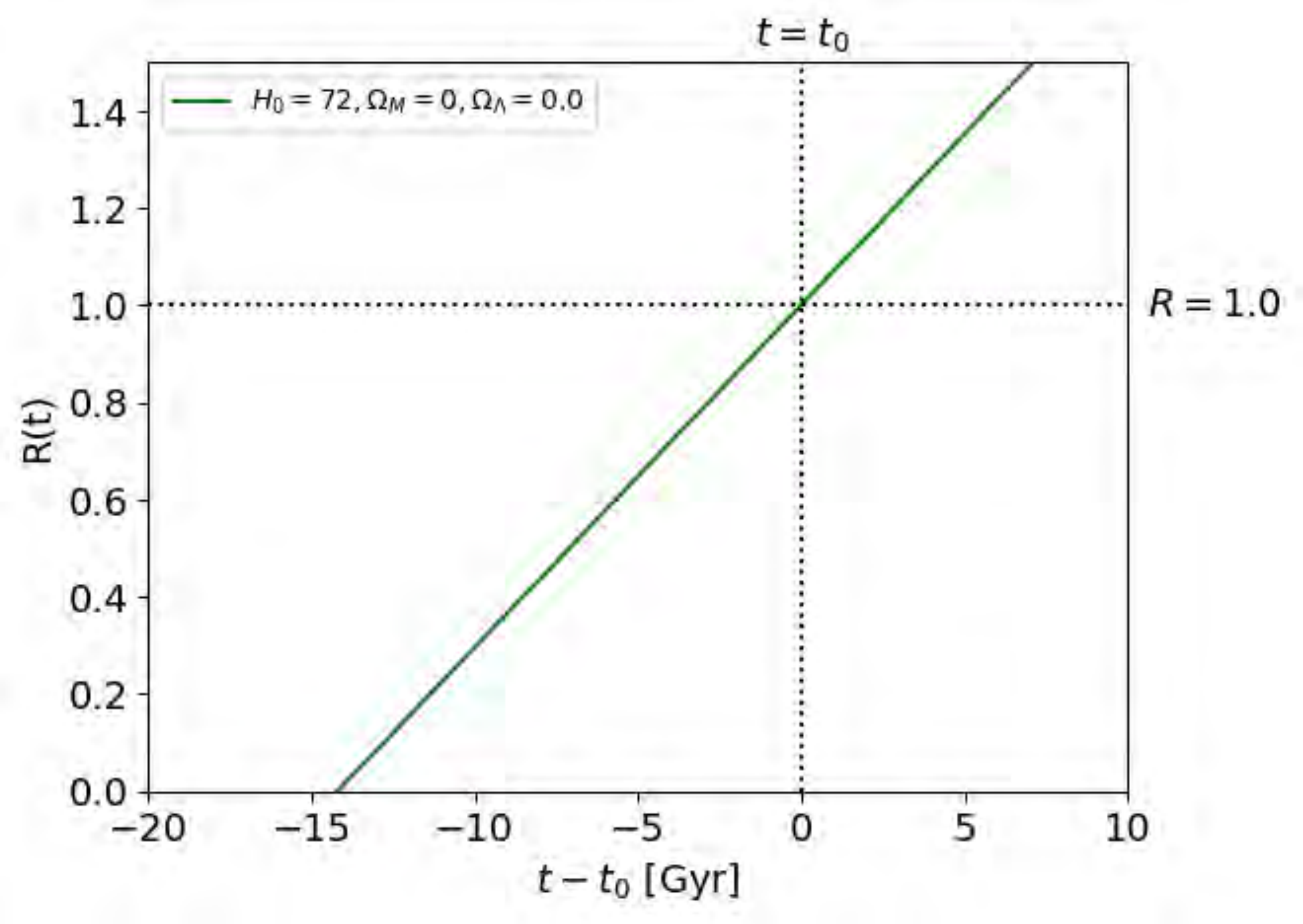


The R(t) plot: Understanding the parameters graphically and intuitively

Now add matter to the Universe (so $\Omega_m > 0$) but no cosmological constant ($\Omega_\Lambda = 0$). Matter has gravity, and gravity slows down the expansion: deceleration. The Universe must have been expanding faster in the past, so R(t) must be "bending downwards."

But we still have to obey the boundary conditions!

- $R(t_0) = 1$
- $H(t_0) = \left(\frac{\dot{R}(t_0)}{R(t_0)}\right) = \dot{R}(t_0) = H_0$



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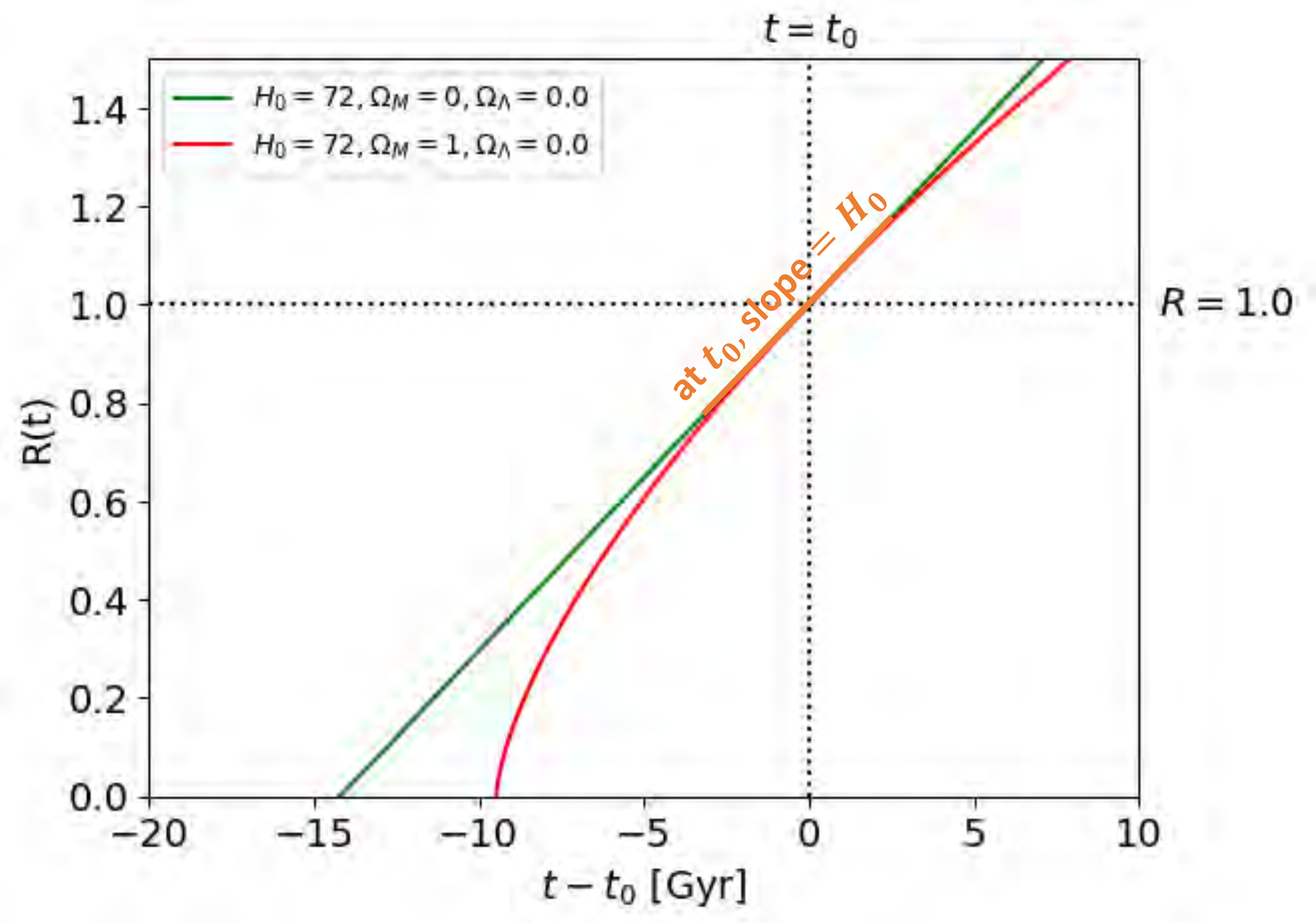
- $R(t_0) = 1$
- $H(t_0) = \left(\frac{\dot{R}(t_0)}{R(t_0)}\right) = \dot{R}(t_0) = H_0$

The more mass you put in (bigger Ω_m) the more the curve bends (deceleration due to gravity is getting strong), and the younger the Universe gets.

For $\Omega_m = 1, \Omega_\Lambda = 0, \dot{R}(\mathrm{t}) \longrightarrow 0$ as t $\longrightarrow \infty$



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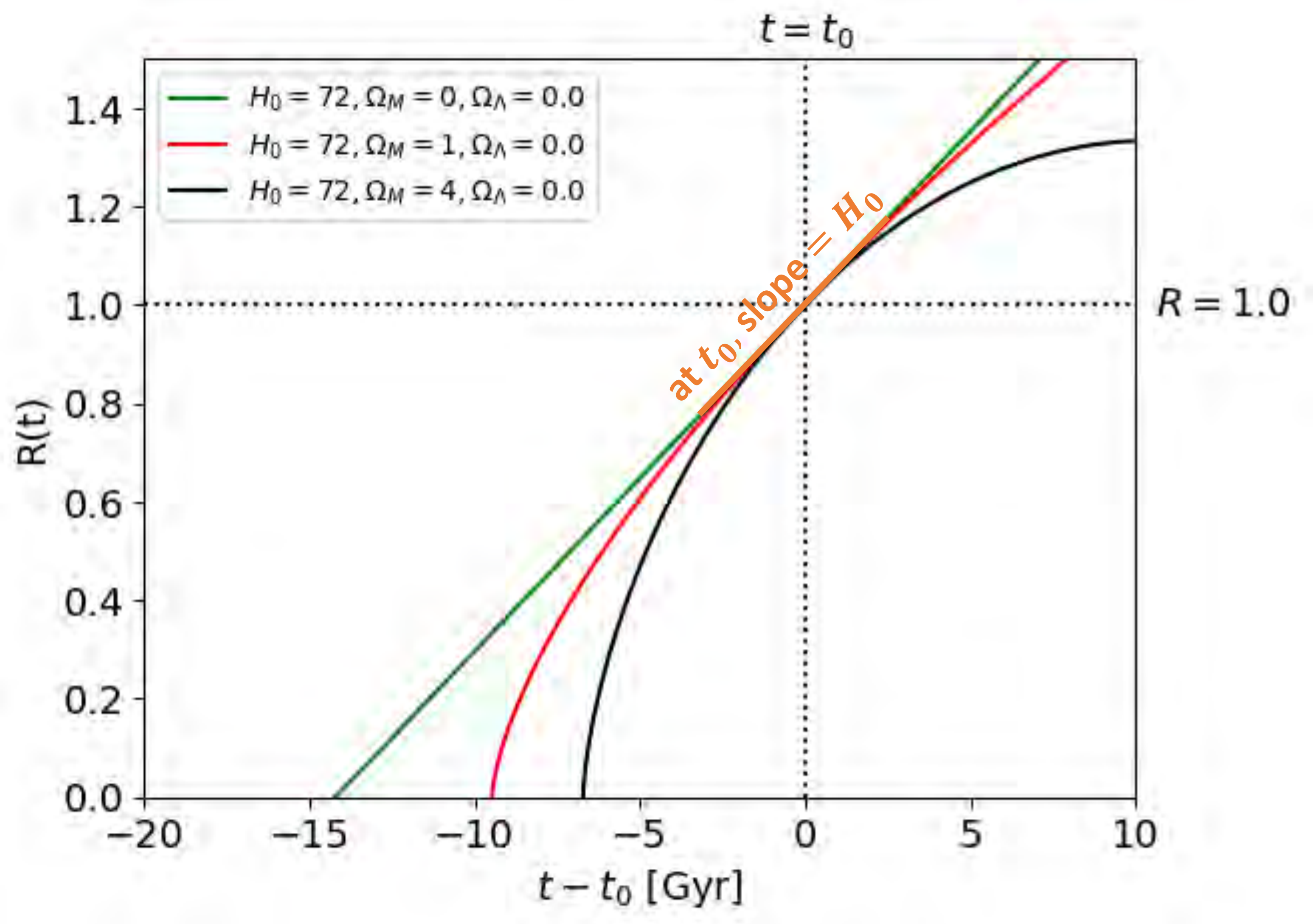
The more mass you put in (bigger Ω_m) the more the curve bends (deceleration due to gravity is getting strong), and the younger the Universe gets.

For $\Omega_m = 1, \Omega_\Lambda = 0, \dot{R}(\mathrm{t}) \longrightarrow 0$ as t $\longrightarrow \infty$

For $\Omega_m > 1, \Omega_\Lambda = 0$, the Universe eventually recollapses.



The R(t) plot: Understanding the parameters graphically and intuitively

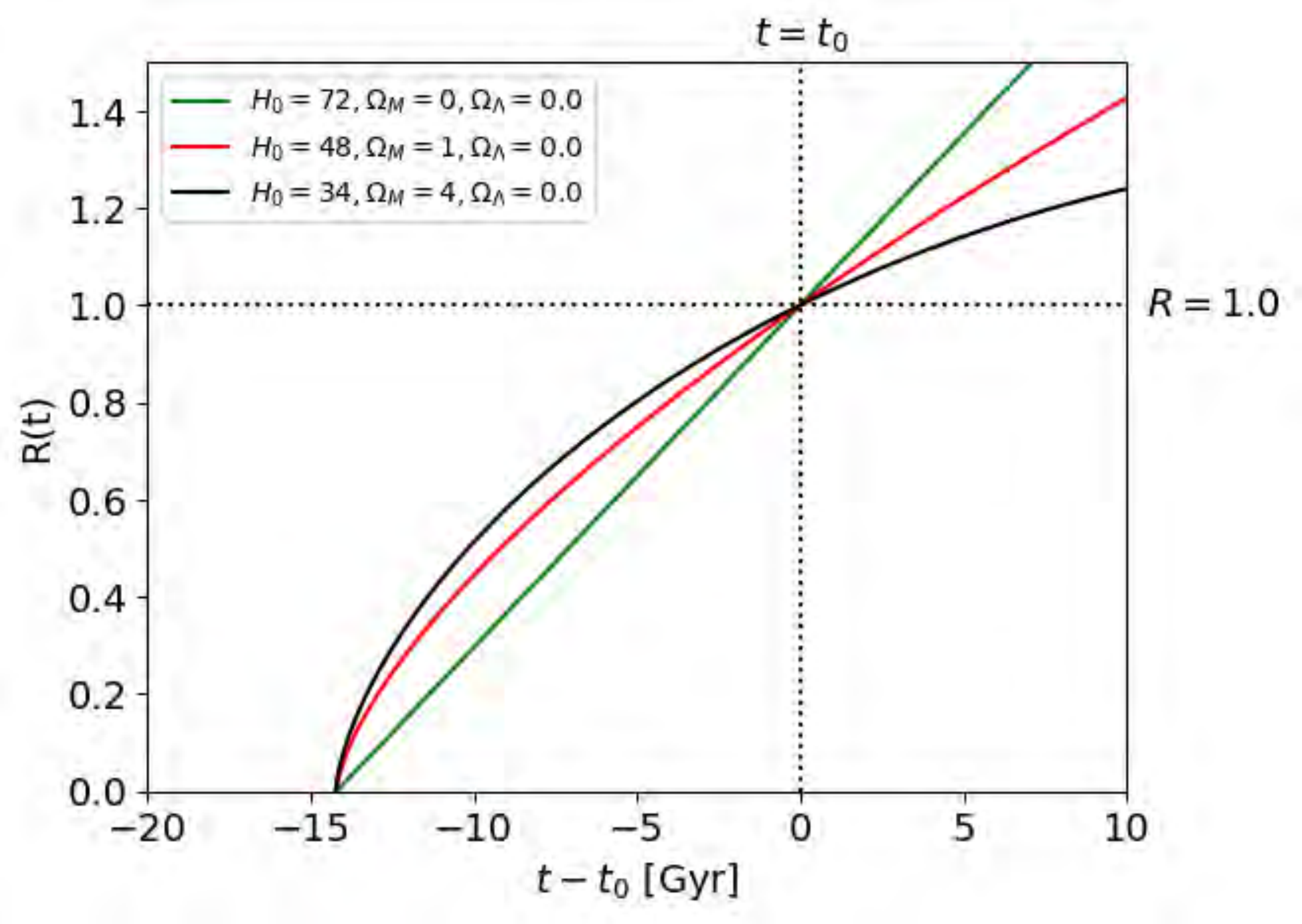
Now add matter to the Universe (so $\Omega_m > 0$) but no cosmological constant ($\Omega_\Lambda = 0$). Matter has gravity, and gravity slows down the expansion: deceleration. The Universe must have been expanding faster in the past, so R(t) must be "bending downwards."

What if we want the same age?

If I wanted to add mass but keep the age the same, I have to change the Hubble Constant H_0. So I'm changing the slope at $t = t_0$.

Three universes with different amounts of matter but having the same age due to different Hubble Constants. ⇒

But we have observational constraints on the Hubble Constant, so we are not free to do just anything we want with it!

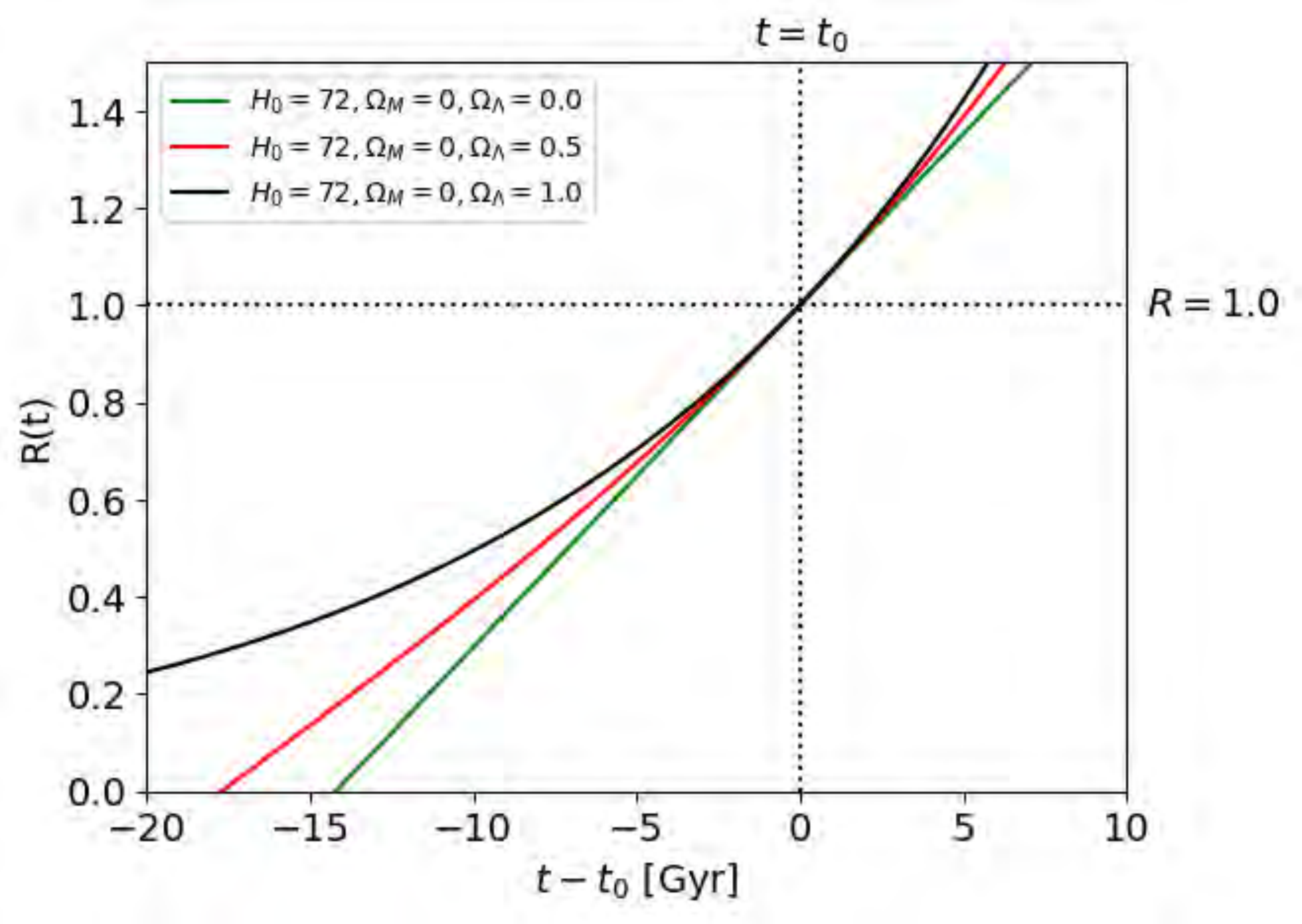


The R(t) plot: Understanding the parameters graphically and intuitively

Now remove matter and add a cosmological constant to the Universe (so $\Omega_m = 0,\ \Omega_\Lambda > 0$). This accelerates the expansion of Universe. It must have been expanding slower in the past, so R(t) must be "bending upwards."

Again, since we have to obey the boundary conditions (R_0, H_0), an accelerating universe must be older than a constant expansion universe.

In fact, if we add too much lambda, we run into the problem of a universe that never has a beginning!

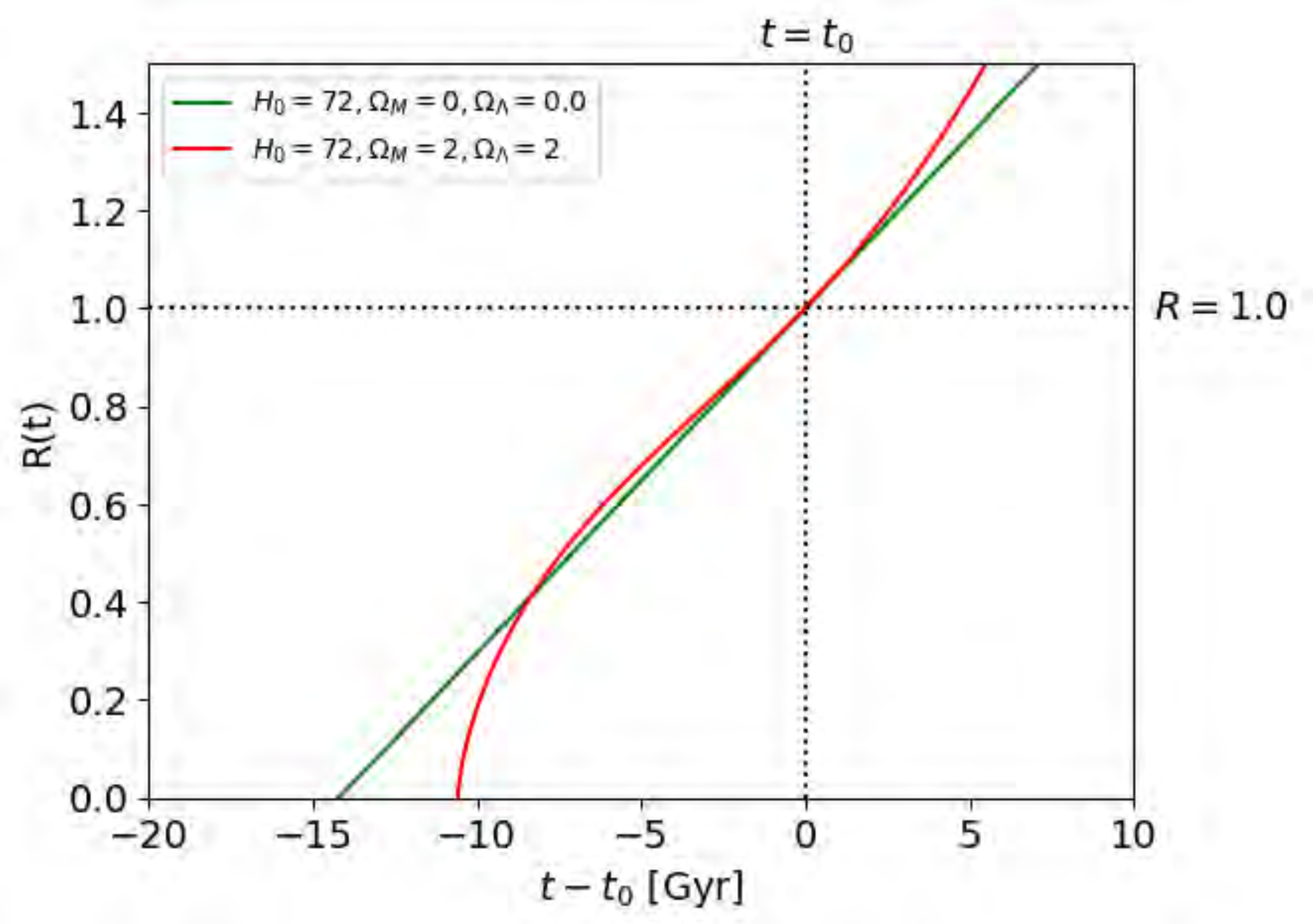


The R(t) plot: Understanding the parameters graphically and intuitively

What if we have both matter and a cosmological constant? So $\Omega_m > 0$ ***and*** $\Omega_\Lambda > 0$. Now we have a competition between matter decelerating the Universe and lambda accelerating the Universe. Who wins?

Go back to the Friedmann Equation:

$$\left(\frac{\dot{R}}{R}\right)^2 - \frac{8}{3}\pi G\rho - \frac{1}{3}\Lambda c^2 = -\frac{kc^2}{R^2}$$



Back in time, the universe was smaller, the density was higher, and gravity wins: the Universe starts out decelerating.

Over time, the density drops, gravity starts to lose, and lambda starts to dominate: late acceleration.

But remember, we still have to obey the boundary conditions (R_0, H_0)

Universes on the $(\Omega_m, \Omega_\Lambda)$ plane

These different behaviors can be mapped onto an $(\Omega_m, \Omega_\Lambda)$ plane to describe the resulting universes.

Remember the governing equations of cosmology. The **Dynamics Equation**:

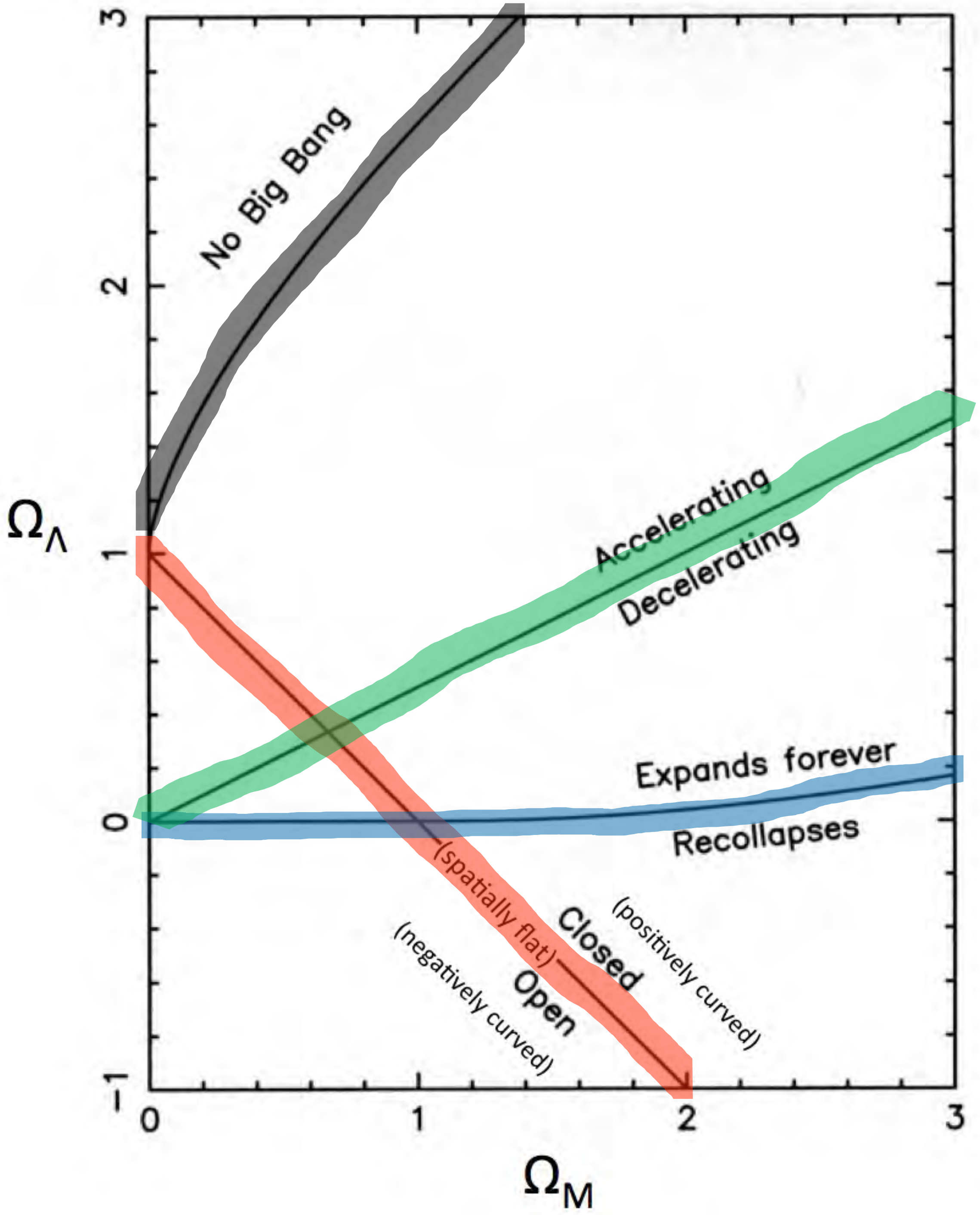
$$\frac{\ddot{R}}{R} = -\frac{4\pi}{3} G\rho + \frac{1}{3}\Lambda c^2$$

The Dynamics Equation shows that Λ (acceleration) works in the **opposite** sense of ρ (deceleration) in determining the expansion history.

The Dynamics Equation solves to the **Friedmann Equation:**

$$\left(\frac{\dot{R}}{R}\right)^2 - \frac{8}{3}\pi G\rho - \frac{1}{3}\Lambda c^2 = -\frac{kc^2}{R^2}$$

The Friedmann Equation shows that Λ and ρ work **together** in determining the curvature.

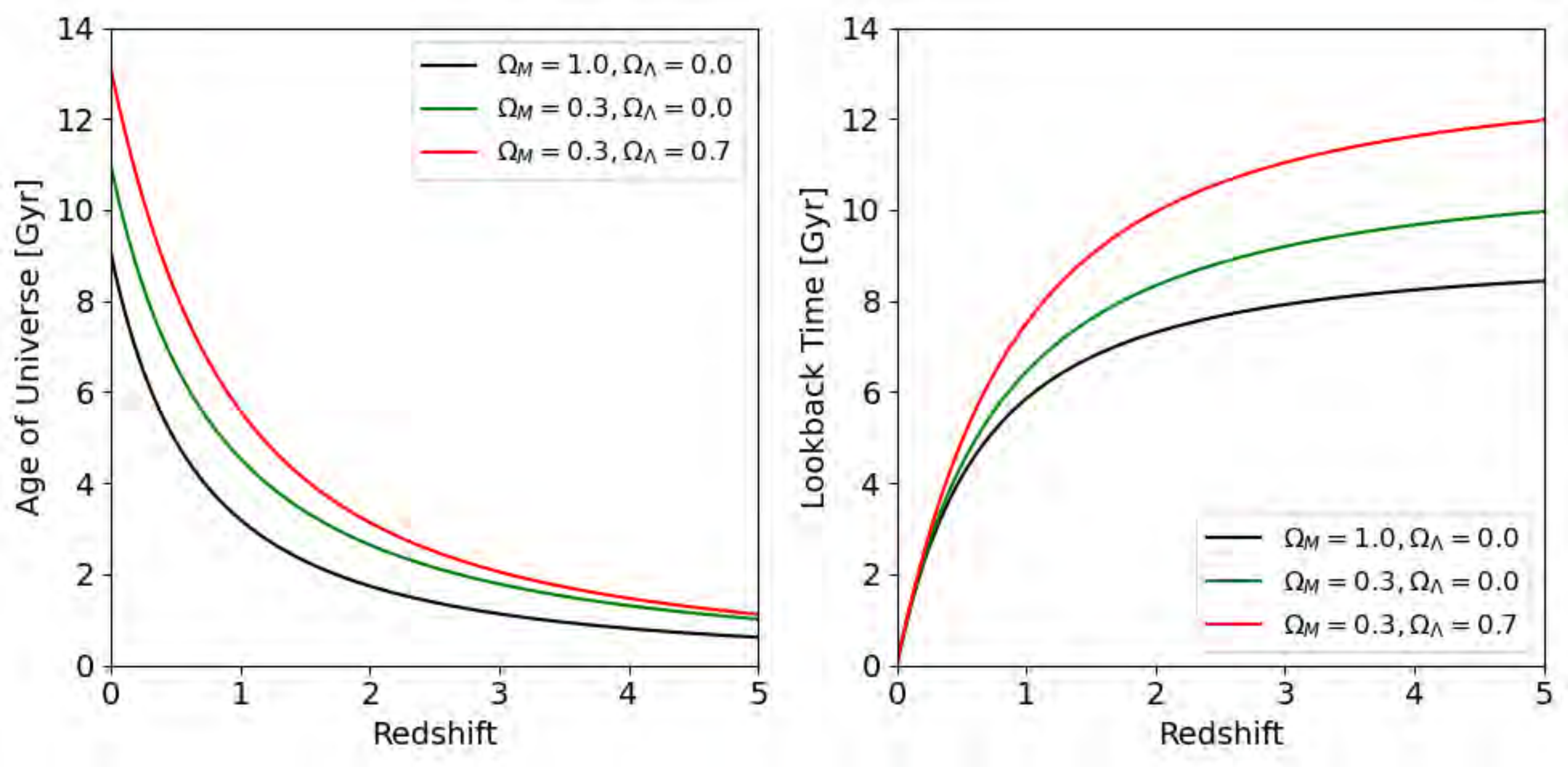


Ages and Lookback times

In all universes, redshift tells you the relative size of the Universe at that redshift: $\mathrm{R} = 1/(1+z)$.
But the connection between redshift and time is different in different cosmologies.

- Age: How old the universe was at a given redshift: $t(z)$
- Lookback time: How far back in the past are we looking at a given redshift: $t_0 - t(z)$

For $H_0 = 72$ km/s/Mpc ⇒



Ages and Lookback times

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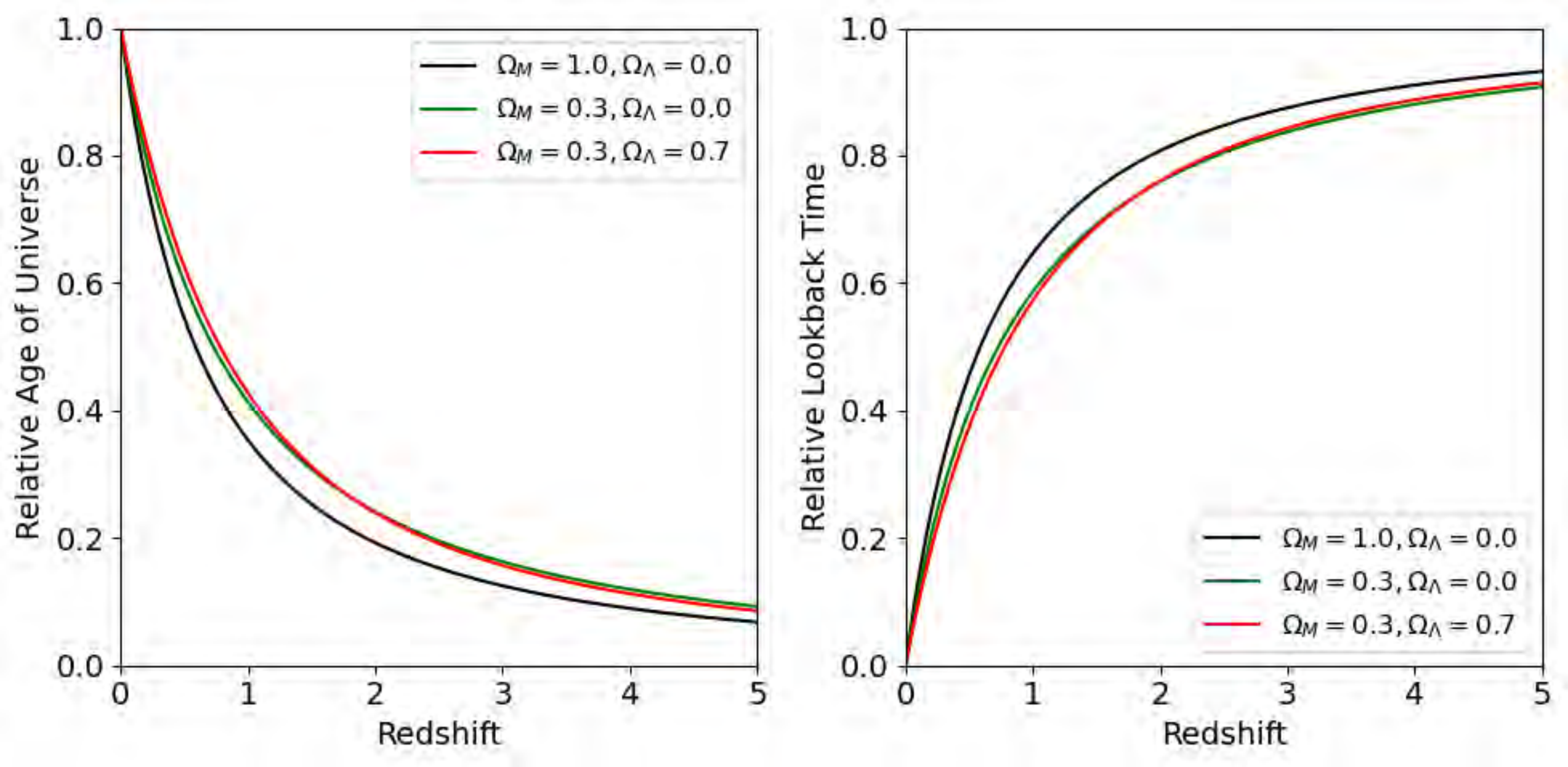
- Age: How old the universe was at a given redshift: $t(z)$
- Lookback time: How far back in the past are we looking at a given redshift: $t_0 - t(z)$

Here it is in relative terms, i.e., fraction of the Universe's current age.

In general:
$z = 1$: ≈ halfway back
$z = 3$: ≈ 85% back

(JWST seeing things at $z > 10$: ≈ 95% back!!!)



The Big Bang

The Friedmann Eqn shows that the universe must be expanding from a very dense and hot initial state.

(Fred Hoyle was skeptical of this notion and in 1949 referred to it derisively in a BBC radio interview as “The Big Bang”. The name stuck.)

Remember, though, the Big Bang is not an explosion of material ***into*** space, its an expansion ***of*** space, carrying material with it.

A dense hot object emits blackbody radiation, which peaks at a temperature given by Wien’s law:

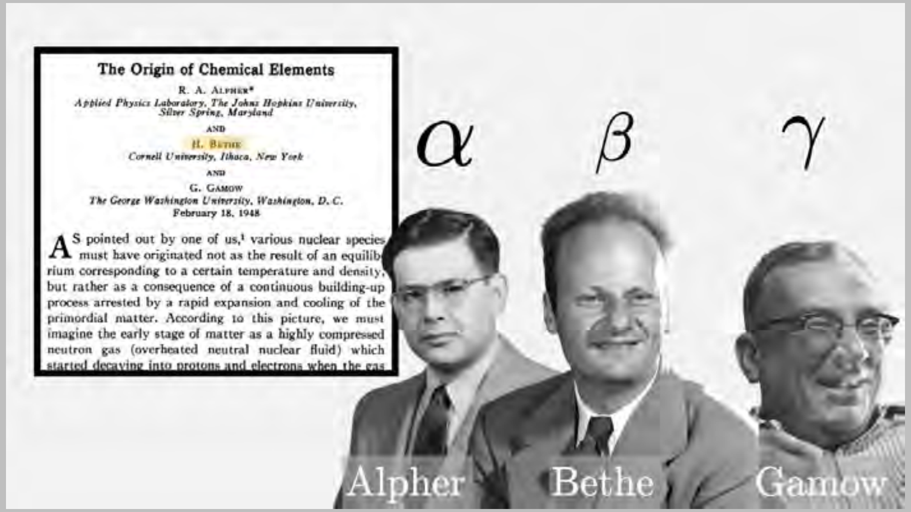
$$\lambda_{peak} = \frac{0.29\ cm}{T\ (K)}$$

As the Universe expands, this blackbody spectrum is redshifted, but keeps the blackbody shape with a temperature that scales inversely with size: $T \sim 1/R$.

We say the Universe cools as it expands, and we should see this redshifted light from the Big Bang coming from all directions.

The Big Bang, nucleosynthesis, and the microwave background

1948: George Gamow and his student Ralph Alpher show that in the early universe the temperature and density would be right to fuse hydrogen into helium, at about the right He:H abundance ratio. They called this **"Big Bang Nucleosynthesis"** and added Hans Bethe to the study, writing the famous "[$\alpha\beta\gamma$ paper]".



Necessary conditions: $T_{BBN} \approx 10^9 K, \rho_{BBN} \approx 10^{-5}$ gm/cm³.

Since density scales inversely with volume ($\rho \sim R^{-3}$), given the current density of the Universe (ρ_0), they worked out that this would happen when the Universe was at a scale factor of

$$R_{BBN} \approx (\rho_0/\rho_{BBN})^{1/3} \approx 3\times10^{-9}$$

And if $T \sim 1/R$, the Universe today should have a temperature of about

$$T_0 \approx T_{BBN}(R_{BBN}/R_0) \approx 3 \text{ K}$$

A 3K blackbody peaks at microwave wavelengths, so today's universe should be bathed in **the microwave background**.

The Big Bang, nucleosynthesis, and the microwave background

So in 1948, the microwave background was predicted but not yet observed.

Early 1960s: Princeton scientists were developing a new microwave/radio observatory to search for these cosmic microwaves, when suddenly....

...they get a call from AT&T Bell Laboratories.

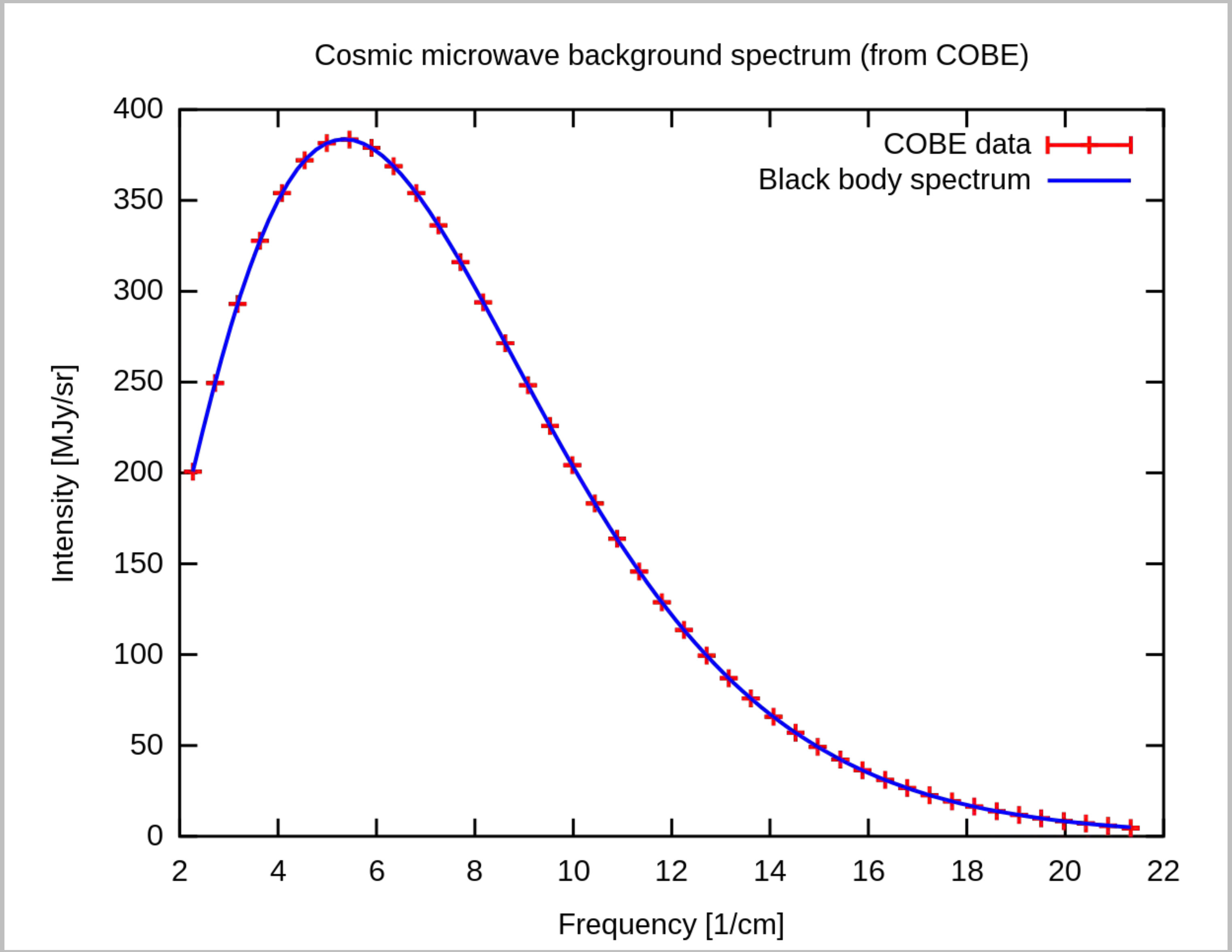
1964: Arno Penzias and Robert Wilson

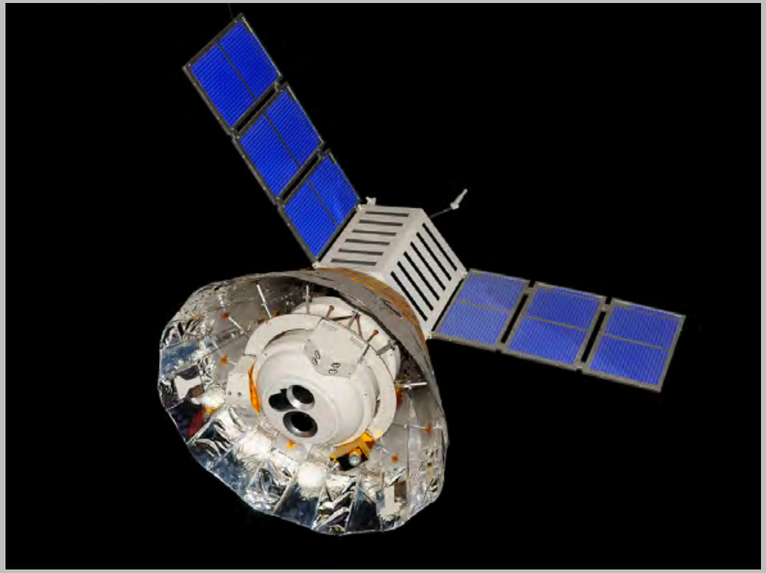
Bell Lab engineers working on a radio antenna to communicate with the new Telstar satellites.

Report a persistant all-sky “hiss” in their equipment: the discovery of the cosmic micowave background (CMB).

The cosmic microwave background (CMB)

The CMB is a perfect blackbody with a temperature of 2.726 K.



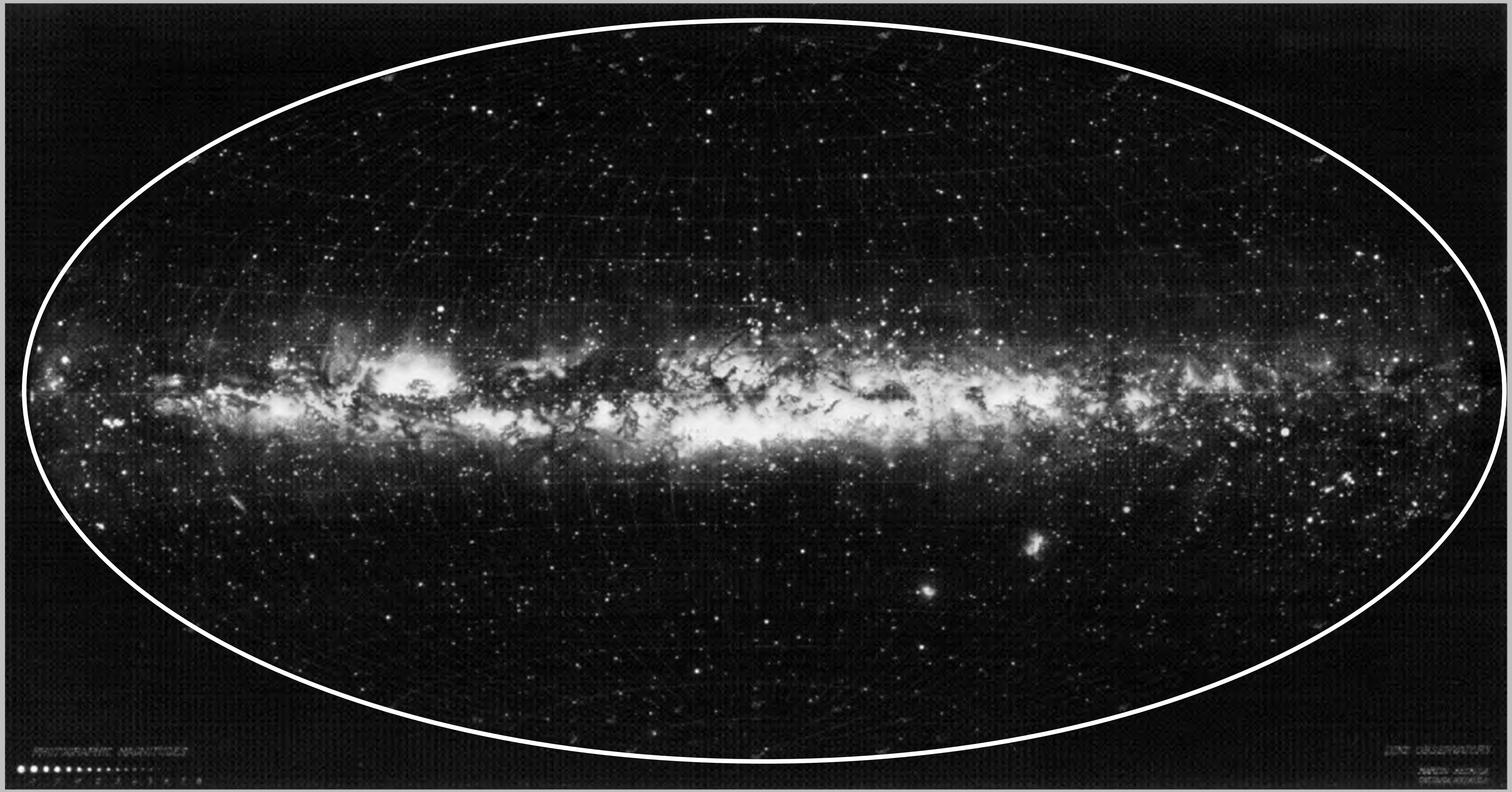


COsmic Background Explorer (COBE)
1989-1993

The cosmic microwave background (CMB)

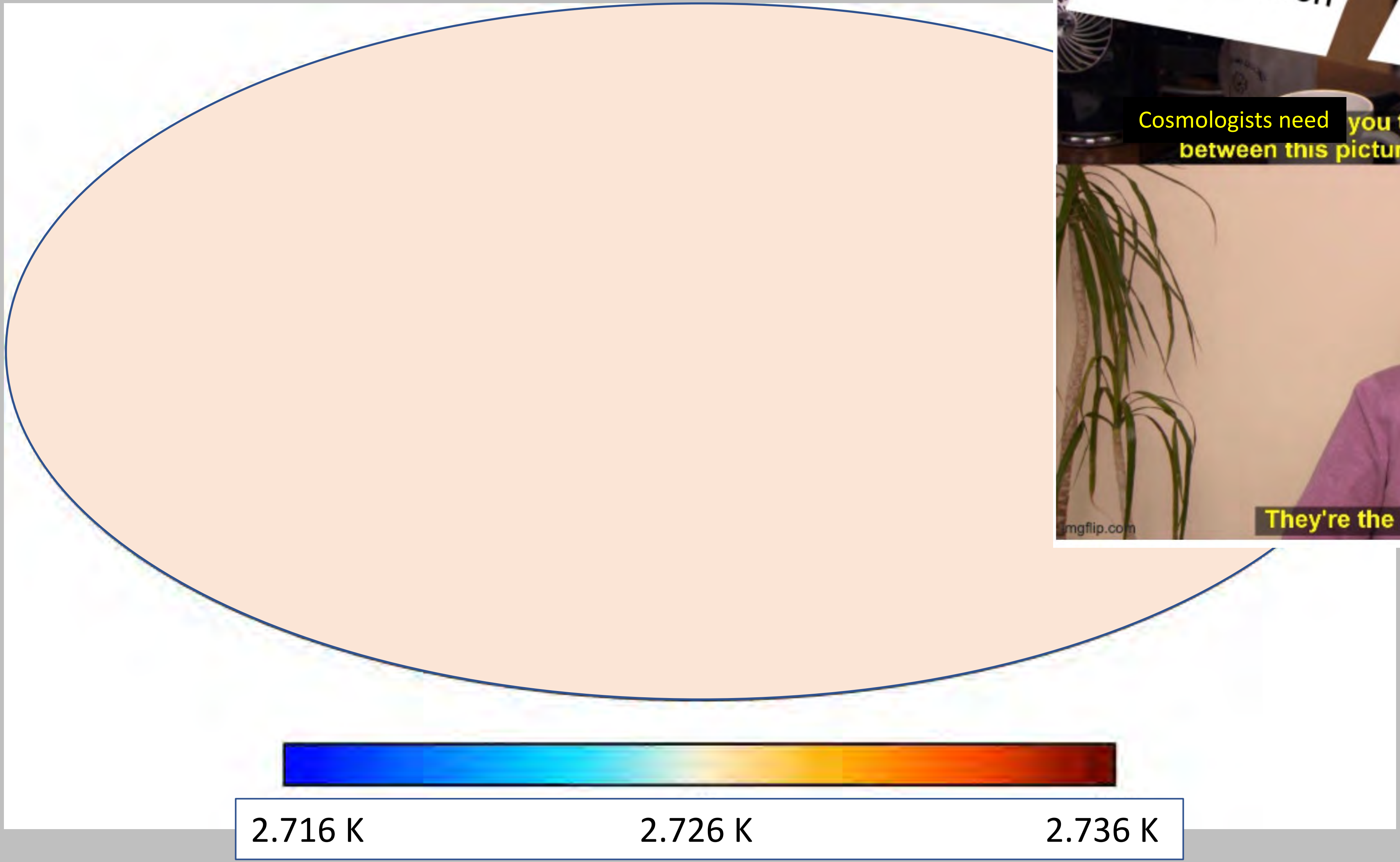
Remember what an all-sky map of starlight looks like....

all-sky optical map



The cosmic microwave background (CMB)

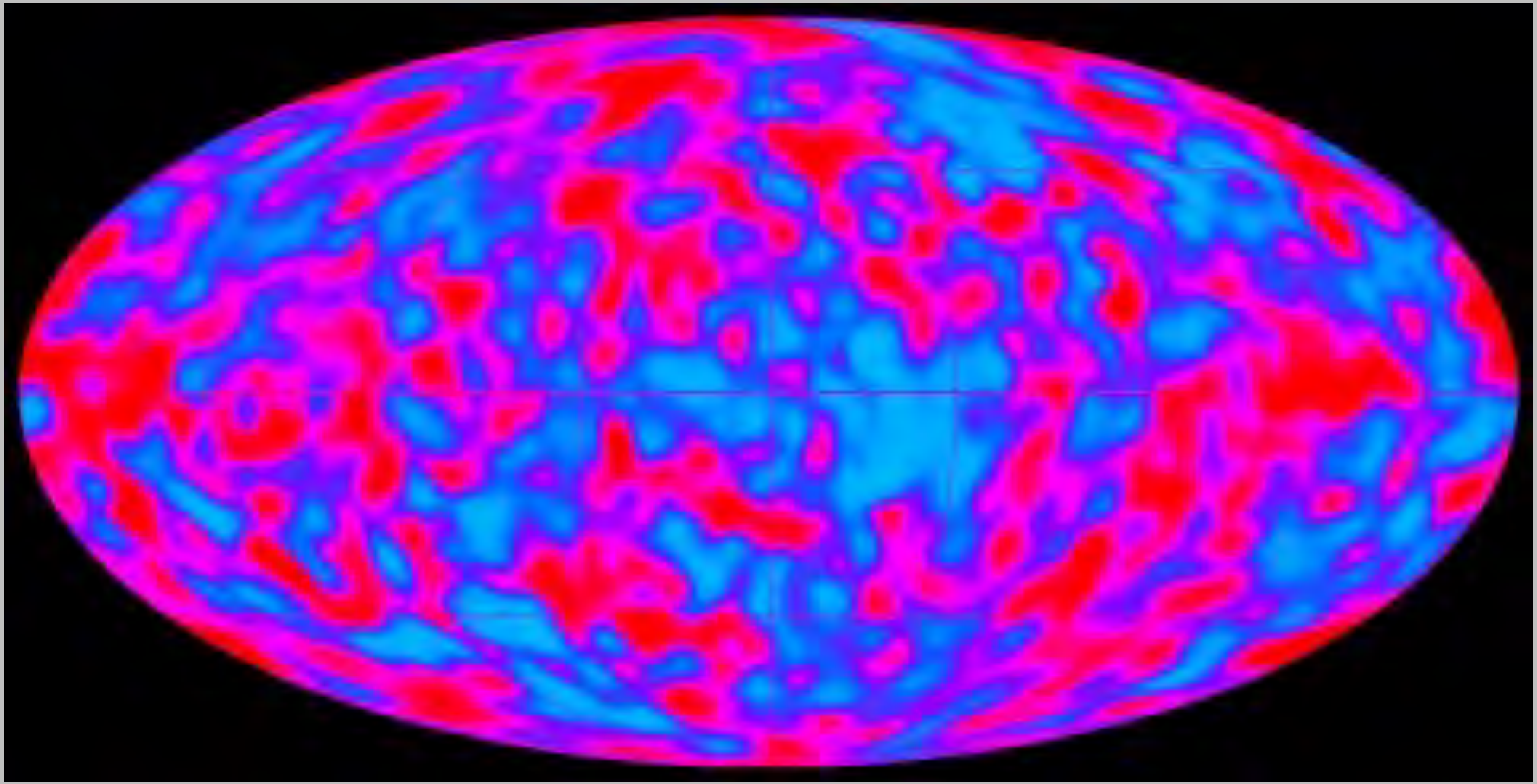
The CMB is a perfect blackbody with a temperature of 2.726 K.
The all-sky CMB map is perfectly smooth (± 0.01 K).



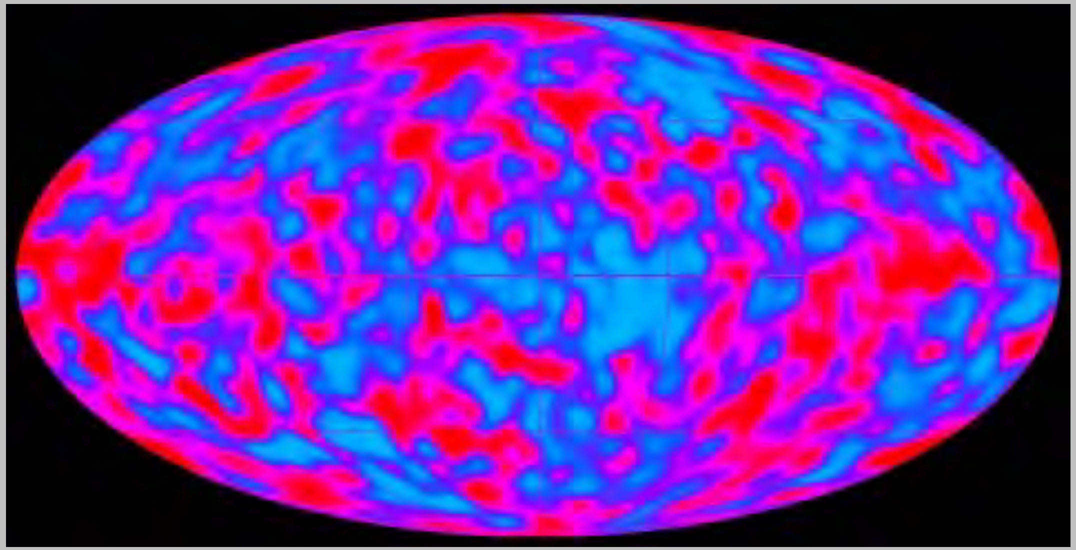
The cosmic microwave background (CMB)

The CMB is a perfect blackbody with a temperature of 2.726 K.
The all-sky CMB map is perfectly smooth (± 0.01 K).
Well, almost perfectly smooth (± 0.0003 K)

COBE all-sky microwave map (1992)



The cosmic microwave background (CMB)



What are we actually seeing?

This is light from the very early Universe, redshifted into microwave frequencies.

In the hot, dense early Universe, everything was ionized, lots of free protons and electrons. Photons couldn't travel very far before being scattered by the electrons, so the Universe was **opaque**.

But the Universe is expanding and cooling, and at some point the temperature drops low enough that electrons and protons can bind together to form bound atoms ("**re-combination**"). At this point there are no more free electrons, and photons can travel freely: the Universe becomes **transparent**.

This happens when the temperature drops to ≈ 3000 K, which corresponds to a redshift of $z \approx 1000$, or an age of ≈ 350,000 years after the Big Bang. We cannot "look back" to earlier times, because the Universe was opaque earlier than this.

The small temperature fluctuations correspond to regions of higher or lower mass density: the "lumps and bumps" of mass that will grow up to be galaxies and galaxy clusters: the large scale structure we see today.

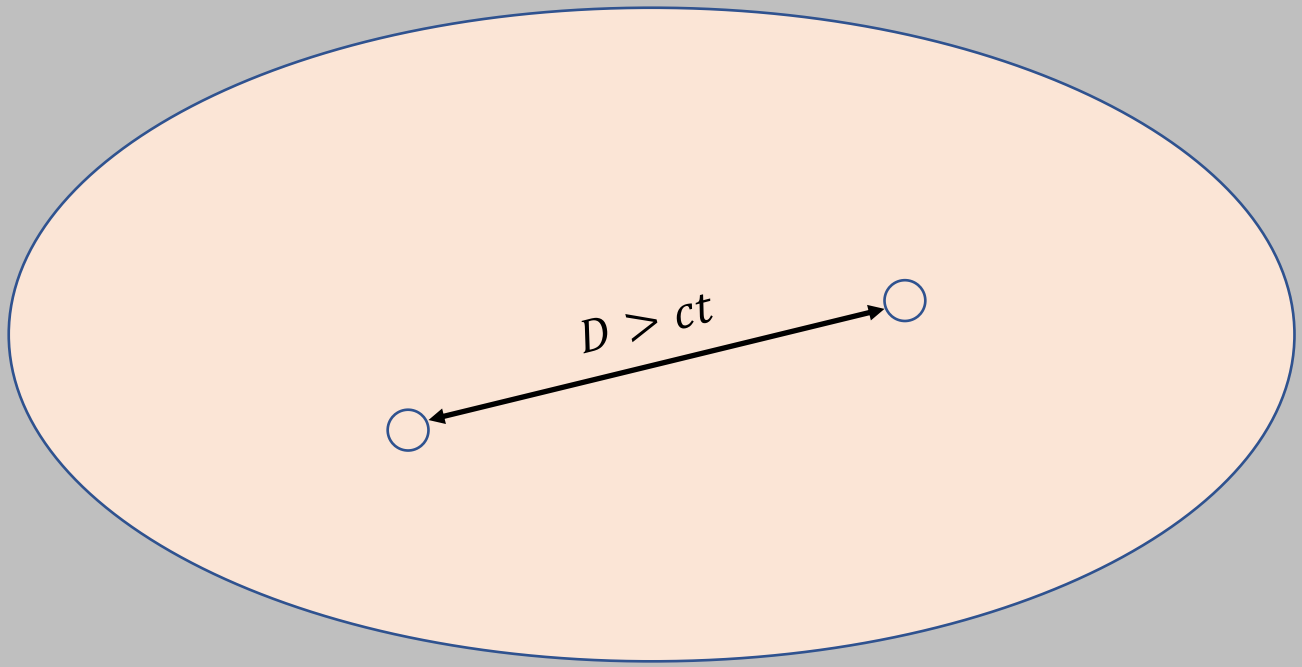
1970s – 80s: The rise of the flat matter-dominated Universe

Cosmologists were faced with several problems, two of which were particularly difficult

The Smoothness Problem: The fact that the temperature of the CMB is so uniform ($\Delta T/T \approx 10^{-5}$) violates causality.

Two widely seperated patches of the CMB were too far apart – even when the Universe was much smaller – to be causally connected at that time: $D > ct$

So how would they know to have precisely the same temperature?



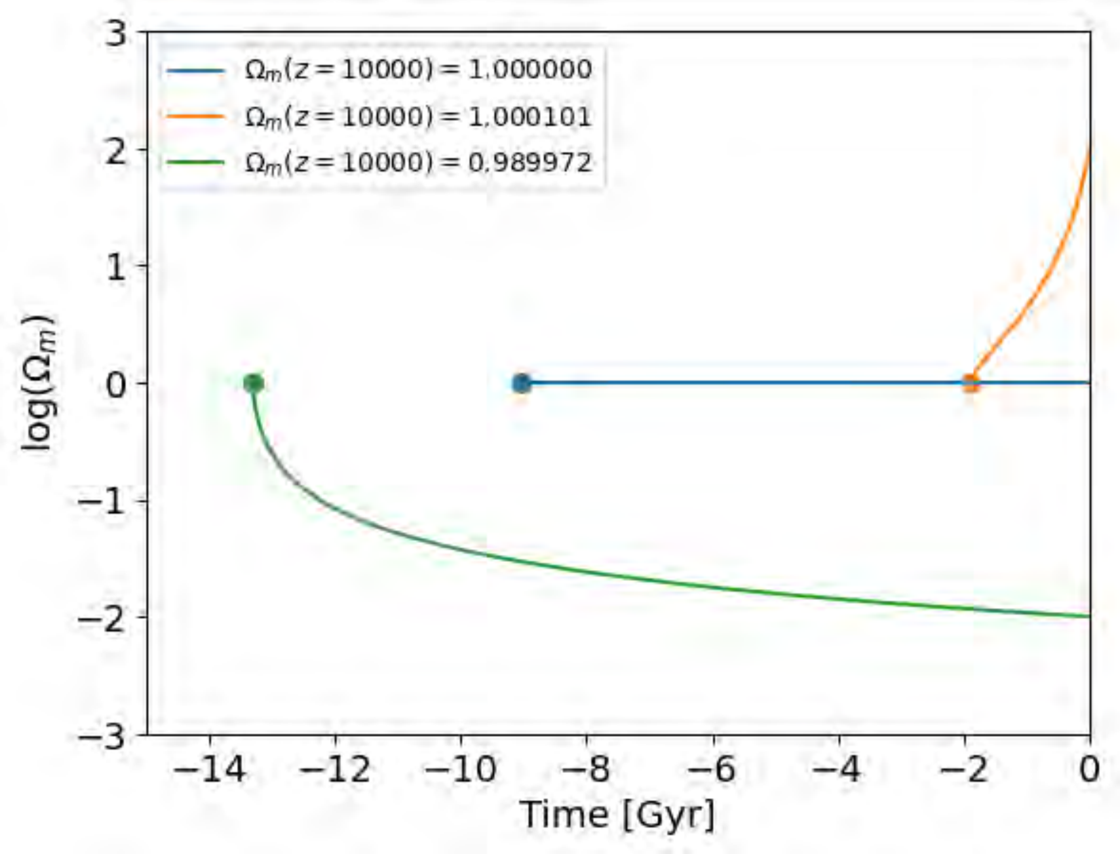
1970s – 80s: The rise of the flat matter-dominated Universe

Cosmologists were faced with several problems, two of which were particularly difficult

The Flatness Problem: The Friedmann equation tells us that Ω_m changes with time as the Universe expands. The only universe that doesn't happen in is an $\Omega_m = 1$ universe.

If Ω_m was *slightly* different from 1 in the early universe, it would be *wildly* different now:

Ω_m @ $z = 10000$	Ω_m today
1.000000	1.0000
1.000101	100
0.989972	0.01



Observational estimates of Ω_m were in the range of 0.3 – 0.8.

The only likely way the Universe would have Ω_m so close to 1 today is for it to have been precisely = 1 at early times.

Why would that be? Why would the universe be so precisely flat?

1970s − 80s: The rise of the flat matter-dominated Universe

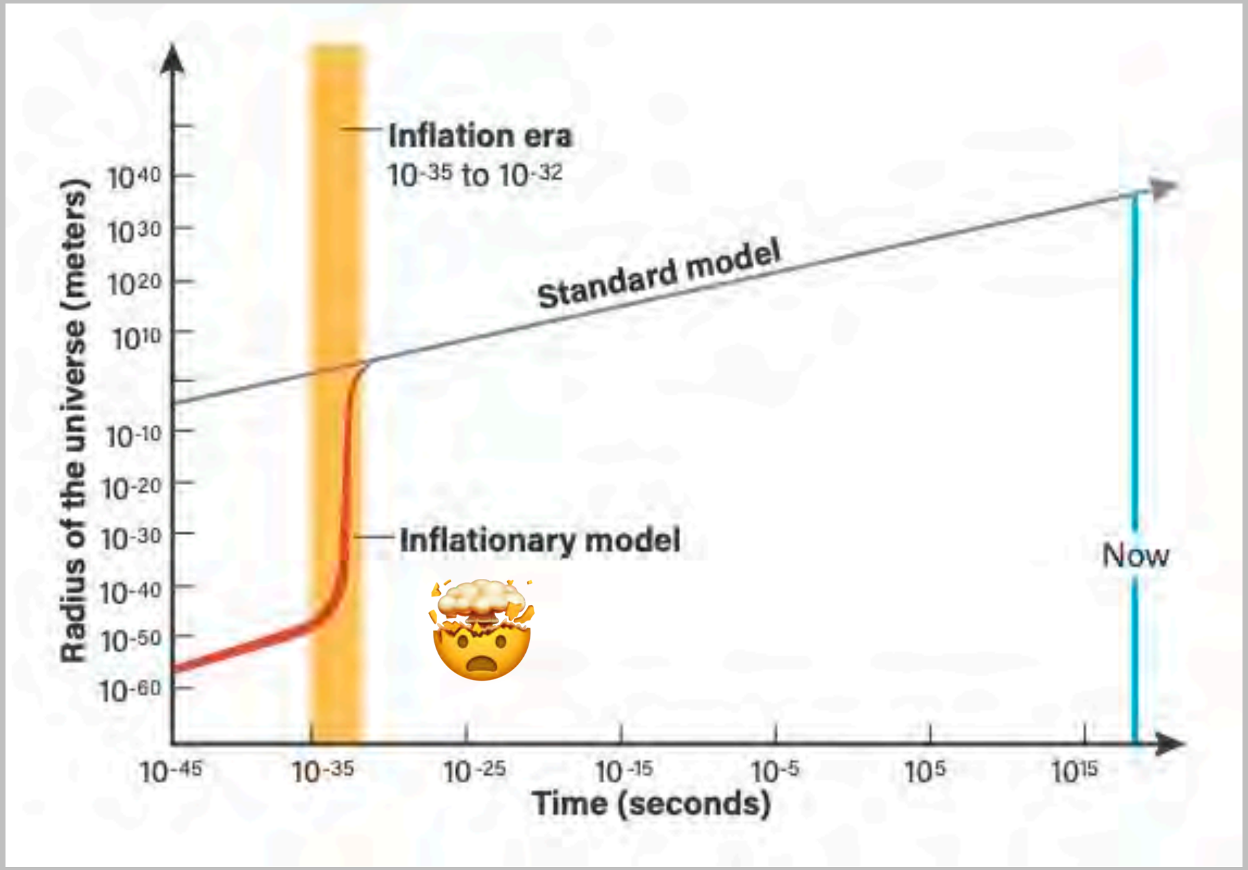
To fix the flat/smooth problems, in the late 1970s the **theory of inflation** was proposed.

In the very early universe, the universe was much smaller than the Friedmann Equation would predict. It was so small that the entire Universe was in causal contact at early times. The entire Universe was homogeneous and smooth.

Then, magically, the Universe inflated at an incredible rate! Those regions that were in causal contact were suddenly inflated so far apart that they are no longer in causal contact.

When? How fast? At $t = 10^{-35}$ seconds, the early Universe inflated by a factor of $\approx 10^{50}$ on a timescale of $\approx 10^{-34}$ seconds.

Why? Who knows? One possibility: this is the moment when strong nuclear force seperated from the electroweak force. This phase transition released energy that drove inflation. But there are other theories, we don't know for sure. *Go ask the physicists.....*



courtesy Astronomy magazine

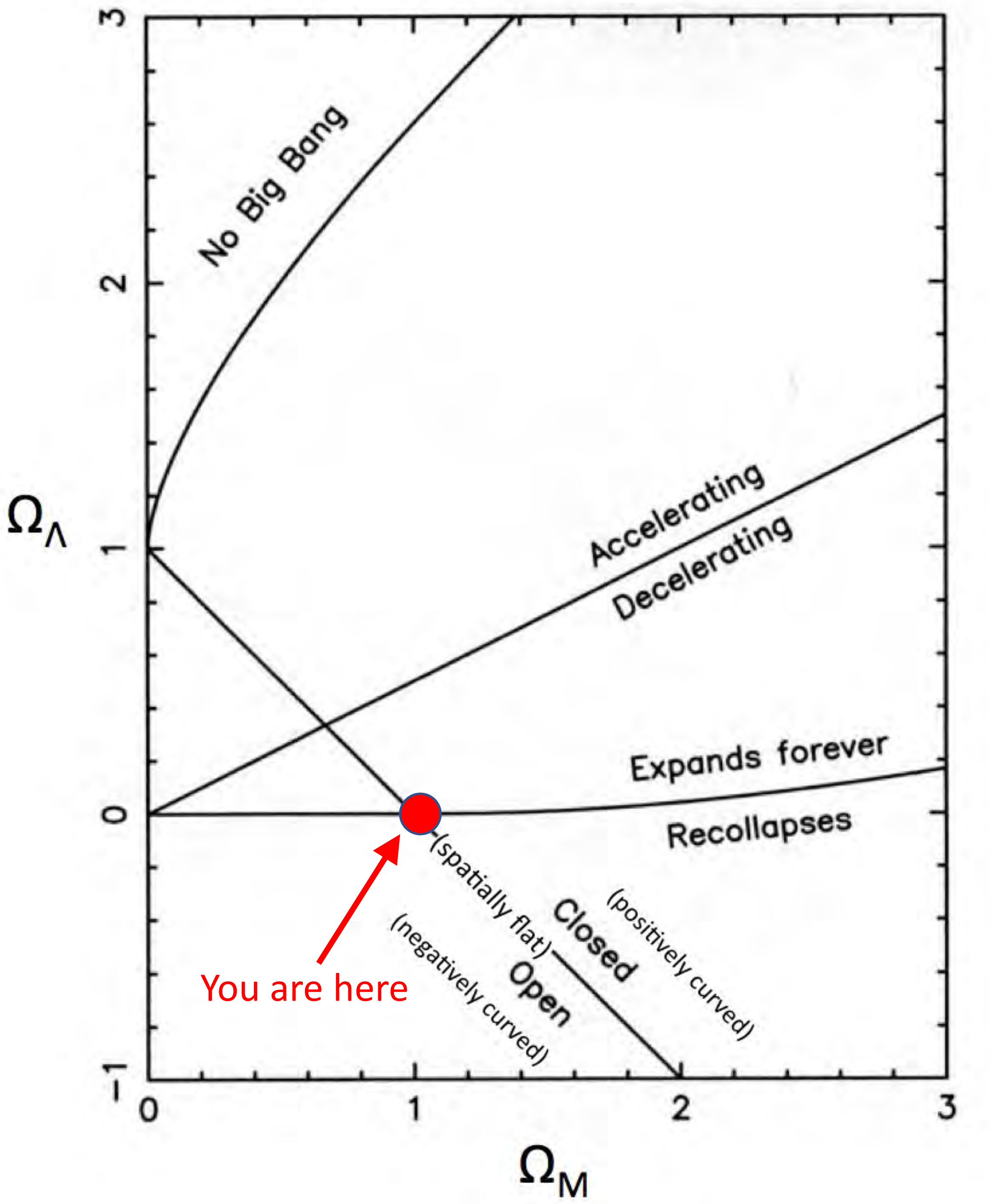
1970s – 80s: The rise of the flat matter-dominated Universe

Inflation fixes many problems:

- **Smoothness**: the early universe was much smaller before inflation, and everything was in causal contact. So no surprise that the CMB has almoste exactly the same temperature everywhere.
- **Flatness**: The inflationary expansion was so big (a factor of 10^{50}) that any curvature is essentially flattened out.

So the natural and expected cosological model was the **Standard Cold Dark Matter** (SCDM) model, as follows:

- The Universe is flat
- The Hubble constant was $H_0 = 70 \pm 20$ km/s/Mpc
- There wasn't much normal matter : $\Omega_b \approx 0.05$ or so
- We knew dark matter existed (galaxy rotation curves, galaxy cluster dynamics, etc), and plausibly would provide enough "missing mass" to get $\Omega_m = 1.0$
- So no need for any crazy cosmological constant: $\Omega_\Lambda = 0.0$

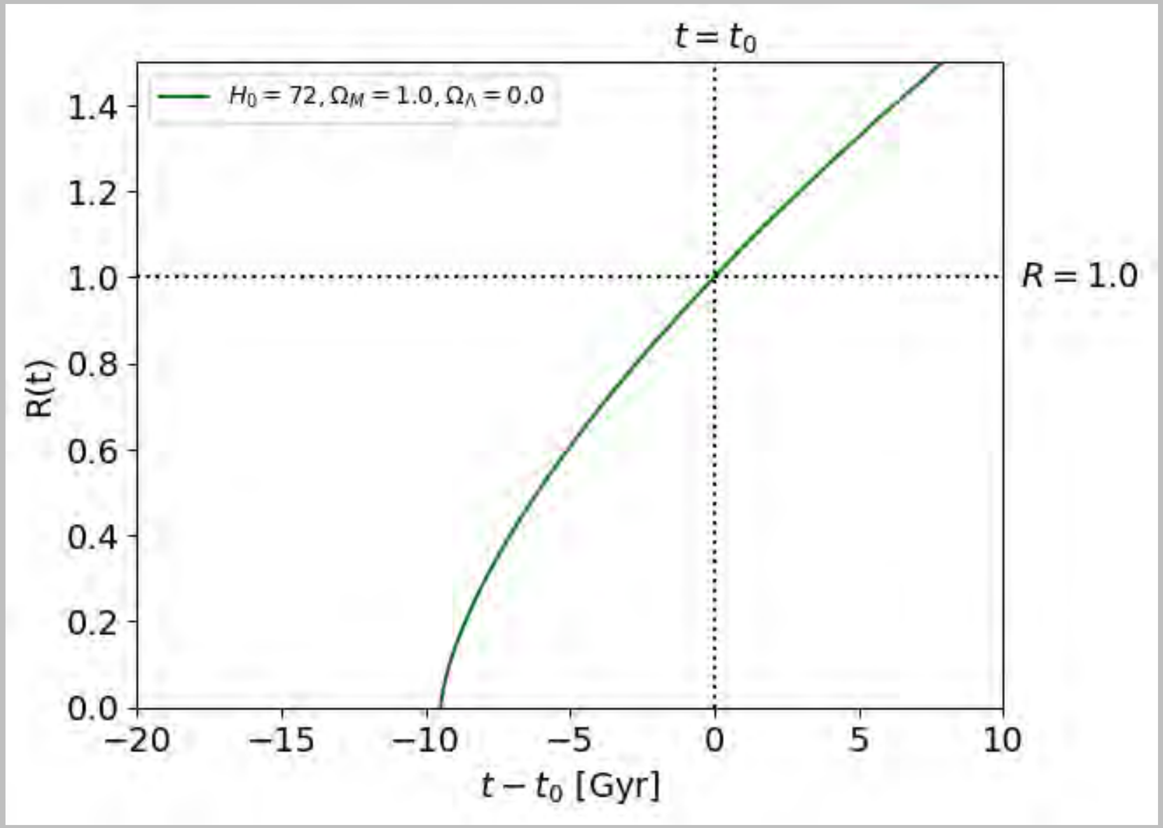


The age of the flat matter dominated universe

Using the Friedmann Equation, we can integrate R(t) for any combination of $H_0, \Omega_m, \Omega_\Lambda$ to work out the age of the Universe. Depending on these parameters, the math can be messy or non-analytic.

$$\left(\frac{\dot{R}}{R}\right)^2 - \frac{8}{3}\pi G\rho \cancel{- \frac{1}{3}}\Lambda c^2 = \cancel{-\frac{kc^2}{R^2}}$$

There is one case in which it is simple – a flat ($k = 0$), matter-only universe: $\Omega_m = 1.0, \Omega_\Lambda = 0.0$



This integrates to

$$R(t) = (6\pi G\rho_c)^{1/3} t^{2/3}$$

Which you can (and will!) solve to get

$$t_0 = \frac{2}{3}\frac{1}{H_0} = 9.3\ Gyr$$

The "Cosmological Crisis" of the early 1990s

By the late 1980s, age estimates for globular clusters were becoming more and more secure: 9 − 12 billion years old.

But the age of a flat, matter-only universe is 9.3 billion years.
How can globular clusters be older than the Universe?

Possibilities to fix this crisis:

- *(Ignore the result)* ***Maybe globular cluster ages are wrong***
- *(Blame someone else)* ***Maybe our estimate of the Hubble constant is wrong***
  If $H_0 = 50$ km/s/Mpc, $t_0 = 13$ billion years.
- *(Believe the astronomical data)* ***Maybe there's less mass*** ($\Omega_\mathrm{m} < 1.0$)
  - If $\Omega_\mathrm{m} = 0.3$, $t_0 = 11.5$ billion years *(barely, maybe works)*
  - If $\Omega_\mathrm{m} = 0.0$, $t_0 = 14.0$ billion years *(ok, that works, but.... no matter of any type?)*
  - *And – ack – the Universe wouldn't be flat!*
- *(Get wild)* ***Maybe we have to consider adding a cosmological constant*** ($\Omega_\Lambda > 0.0$)
  An accelerating Universe is older

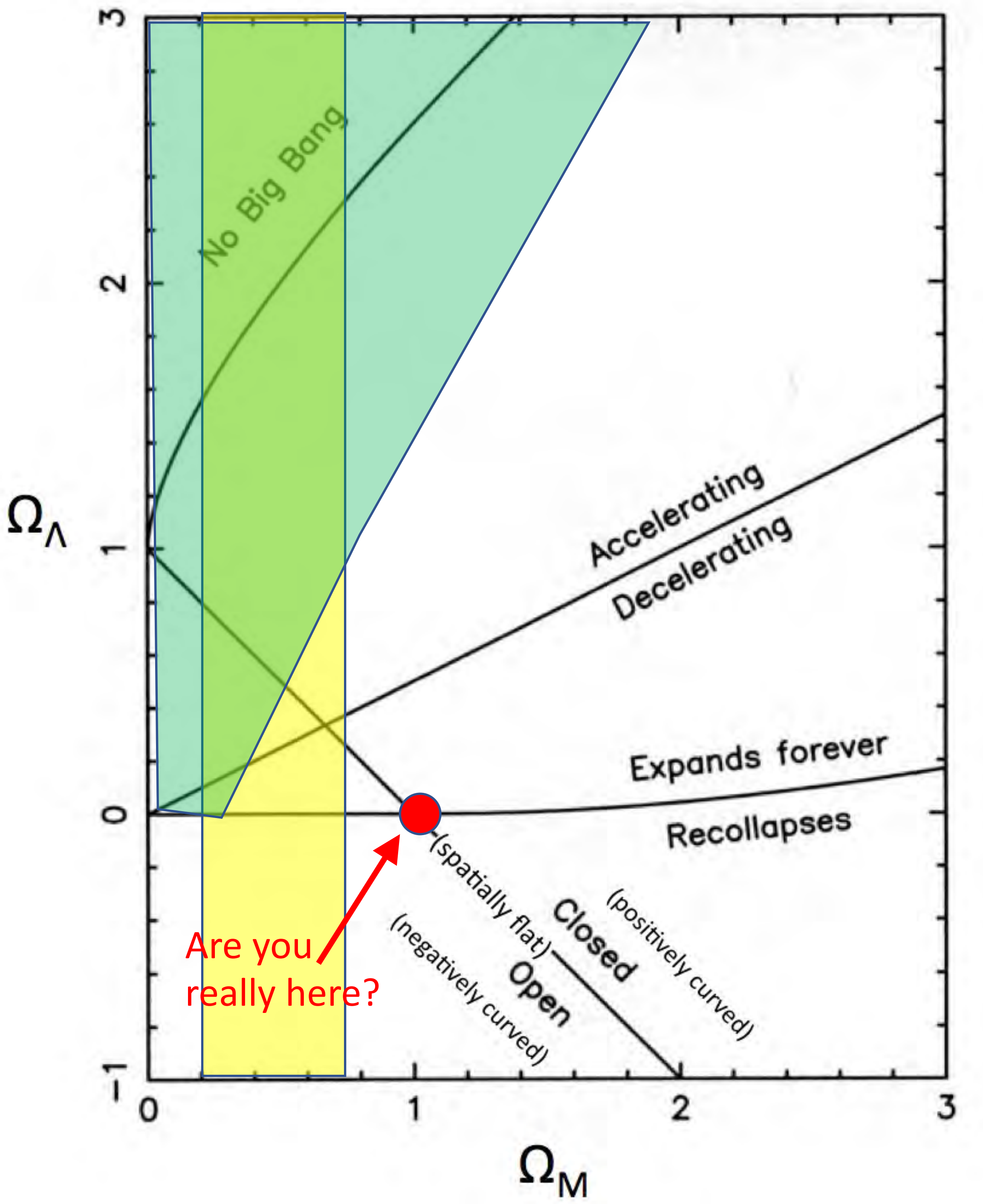
Cosmological parameter constraints

Estimates of H_0 are getting quite accurate, ruling out the low H_0 arguments. Since globular cluster ages are still old

$$\Omega_m \ll 1, \text{ or } \Omega_\Lambda > 0$$

The surveys for matter suggest the universe is less dense than needed to flatten the universe ($\rho > \rho_{crit}$):

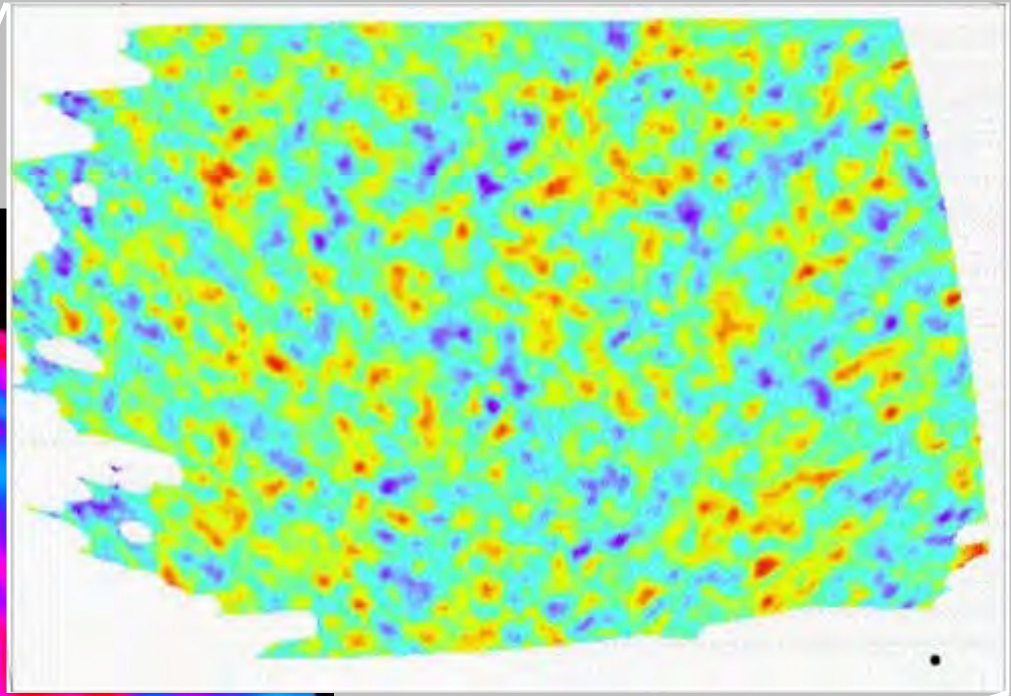
$$\Omega_m \approx 0.2 - 0.7$$



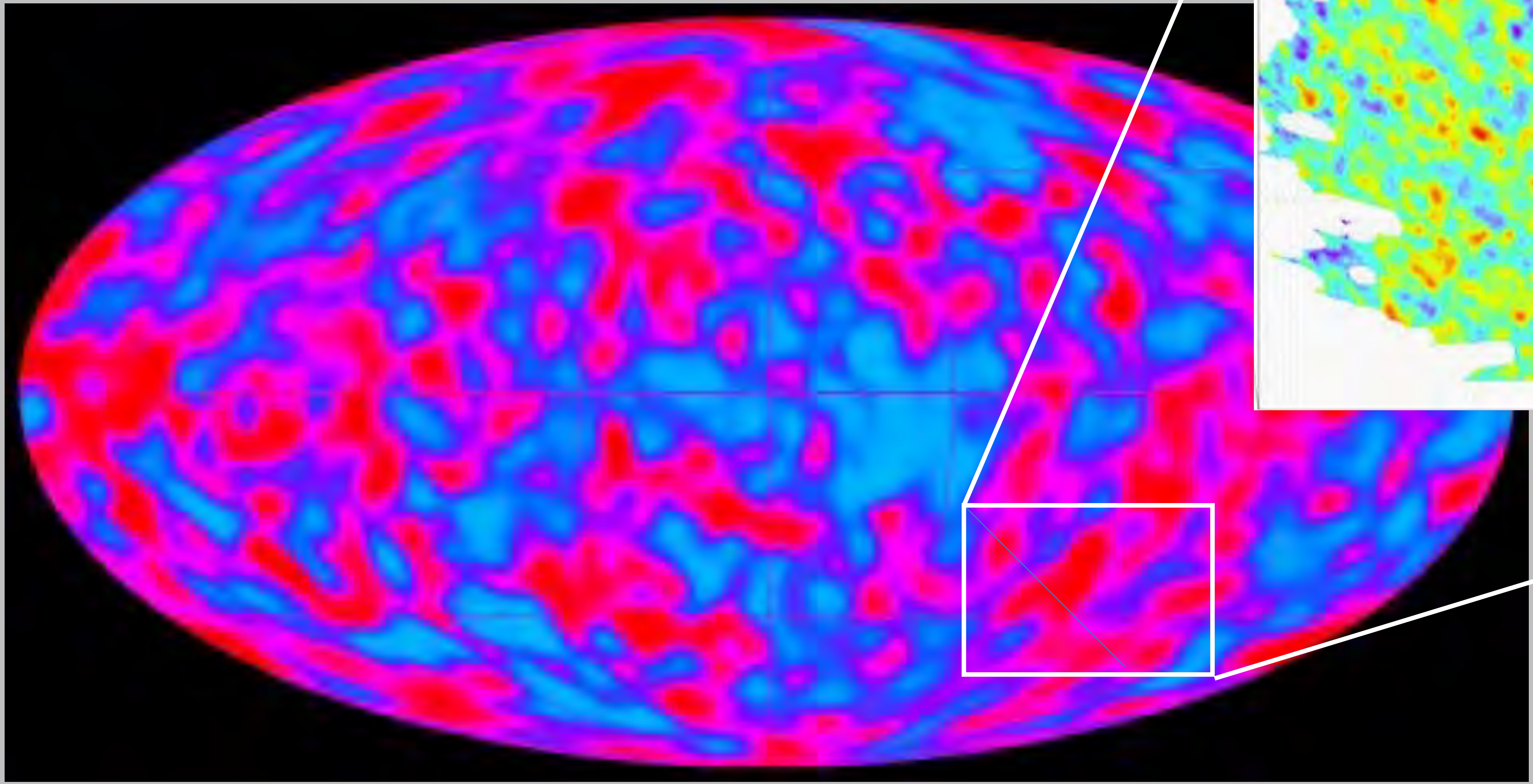
Meanwhile, better data began coming in for the cosmic microwave background (CMB)

Microwave observatories (ground and balloon) began getting images of the microwave background at higher resolution, seeing the temperature fluctuations on smaller scales.

This allowed a new test of the Universe's curvature.



Boomerang experiment (1999)

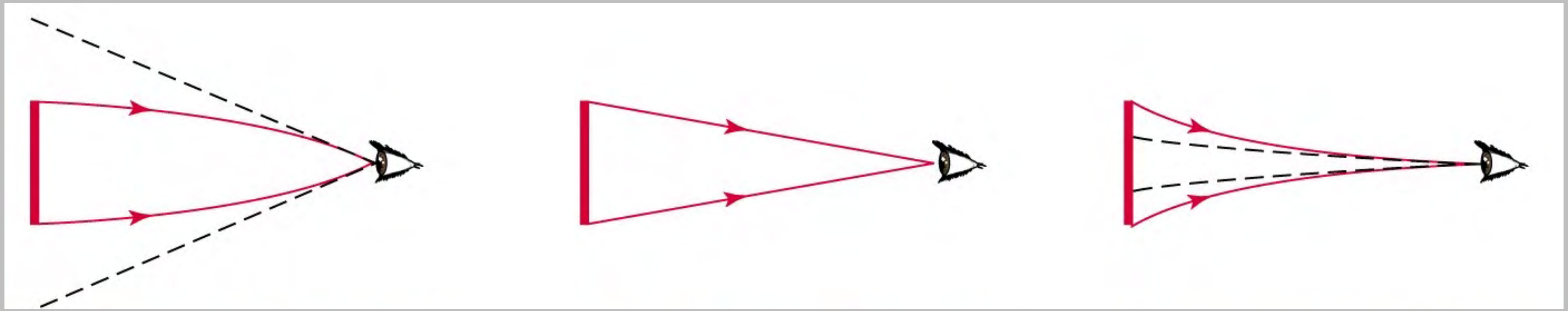


Microwave sky (COBE 1992)

Using the CMB to probe the curvature of the Universe

Imagine looking at an object of fixed physical size (a "standard rod") under different spatial geometries. Since straight lines curve differently under different spatial geometries, an object of fixed physical size will have different angular sizes under different geometries.

positive curvature, larger angular size | spatially flat | negative curvature, smaller angular size



So if you know the physical size of the object, you can predict the different angular sizes for different types of curvature.

In a hot dense medium (like the early universe) pressure waves that grow the overdensities of mass move at the sound speed, which only depends on density and temperature. So the lumps in the CMD will have a characteristic size given by $d = c_s \times t_{CMB} \approx 65$ Mpc in any universe. A standard rod!

c_s : sound speed
t_{CMB} : age of the universe at the time of the CMB

Using the CMB to probe the curvature of the Universe

The observed CMB matches the expectation for a spatially flat Universe!

Spatially flat: $\Omega_\mathrm{m} + \Omega_\Lambda = 1$

So the CMB insists the Universe is flat.

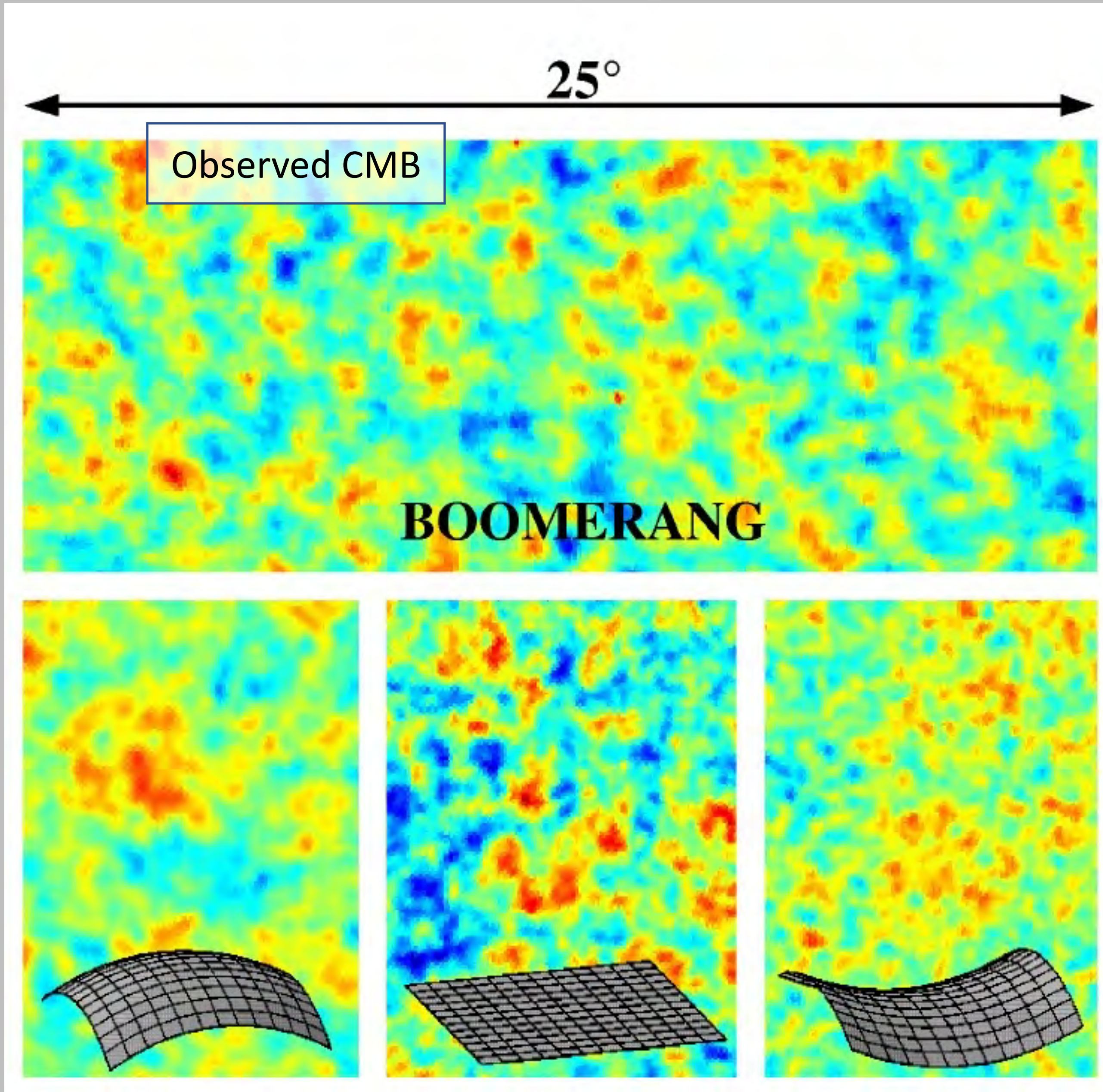
Globular clusters insist the Universe is old.

Surveys of matter in the universe insist $\Omega_\mathrm{m} < 1$.

The cosmological constant is crazy talk.

Something has to give.....

Simulated CMBs under different spatial curvatures ⇒



positive curvature | flat space | negative curvature

Cosmological parameter constraints

Estimates of H_0 are getting quite accurate, ruling out the low H_0 arguments. Since globular cluster ages are still old

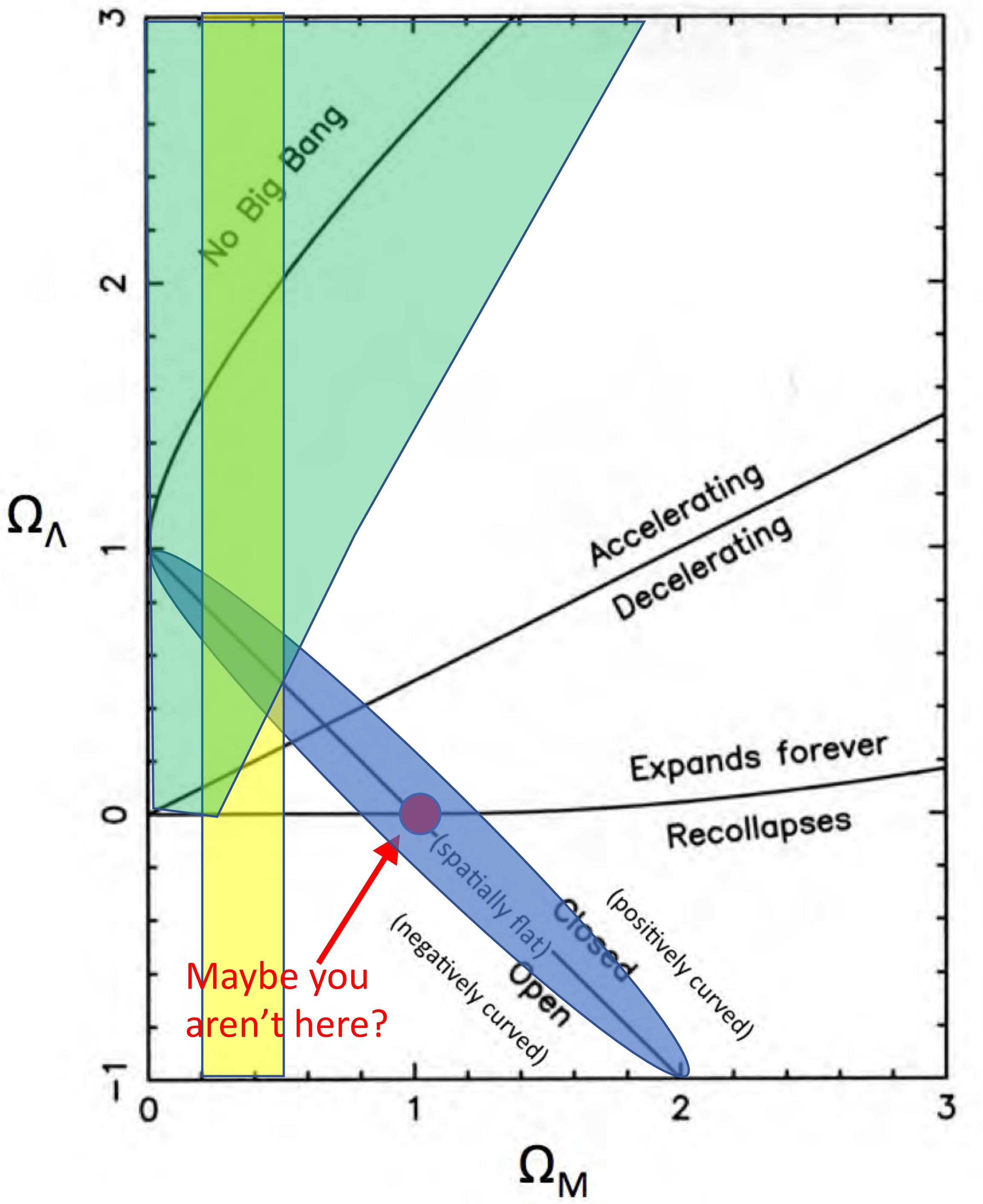
$$\Omega_m \ll 1, \text{ or } \Omega_\Lambda > 0$$

The surveys for matter are getting better, and continue to support a low density universe

$$\Omega_m \approx 0.2 - 0.5$$

The fluctuations in the CMB continue to demand a flat universe

$$\Omega_m + \Omega_\Lambda = 1$$



Measuring the shape of space: the "Redshift-Distance Test"

The apparent brightness of high-redshift objects is different in different cosmologies, due to:

- The curvature of space (the $1/d^2$ effect depends on curvature)
- The expansion history of the Universe (affects how $z \longrightarrow d$)

These can be calculated for difference universes to work out the effective distance modulus.

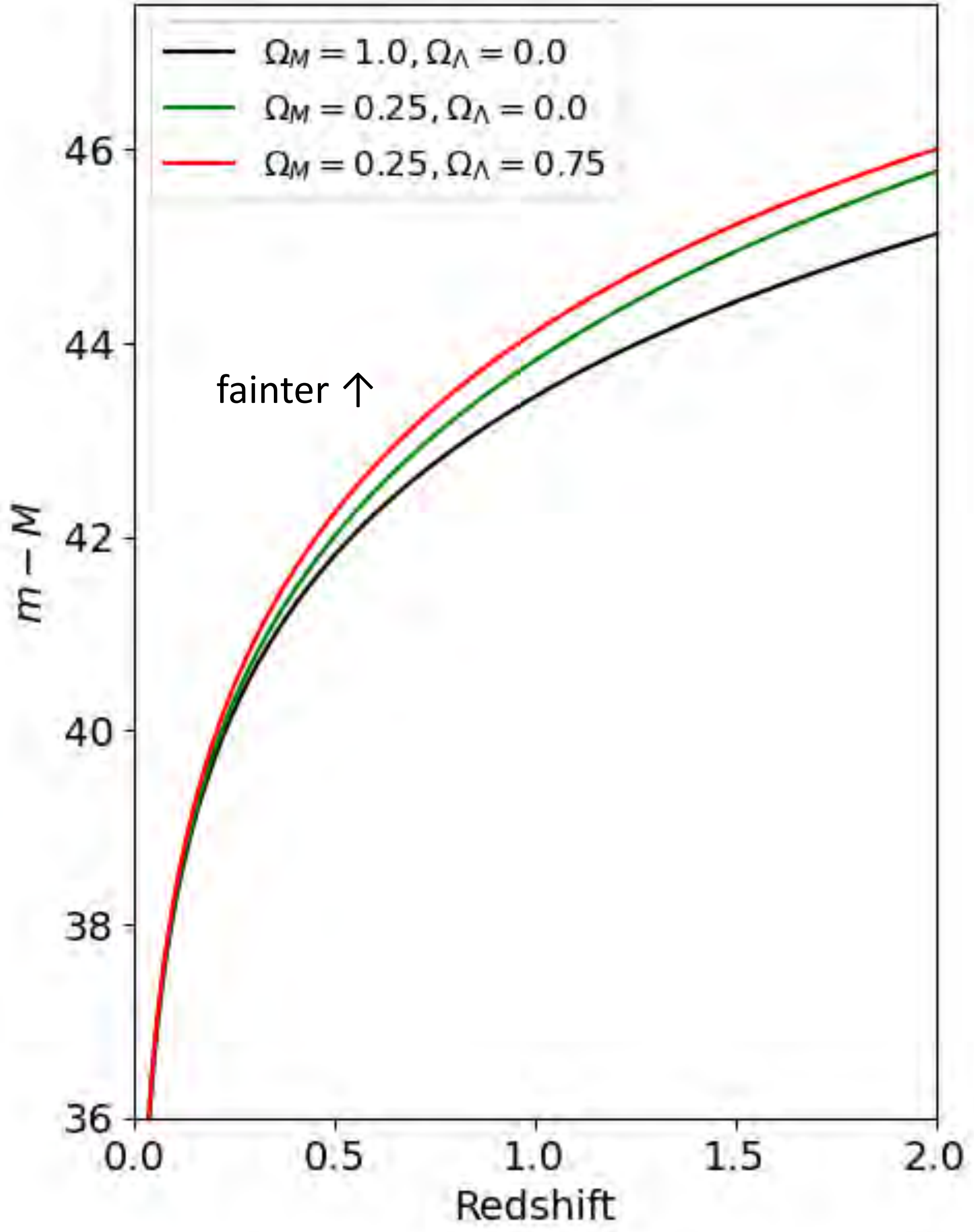
- **locally**, we had: $m - M = 5\log d - 5$
- **cosmologically** we have: $m - M = 5\log D_L - 5$
  where we define Luminosity distance:

$$D_L = f(z, H_0, \Omega_m, \Omega_\Lambda)$$

So if we have a type of object with a fixed, known luminosity (a "standard candle") we can measure its apparent magnitude at different redshifts and see which line it falls on.

Requirements for our standard candle:

- Needs to be a bright object
- Needs to be a precise, fixed luminosity



Type Ia supernovae as standard candles

Remember Type Ia SNe: accreting white dwarfs that detonate when they hit the Chandrasekhar mass of $\approx 1.4\ M_{\odot}$. Their peak magnitude should be similar in all cases.

Host	SN	$M^0_{B,i}$
M101	2011fe	−19.389
N1015	2009ig	−19.047
N1309	2002fk	−19.331
N1365	2012fr	−19.390
N1448	2001el	−19.111
N2442	2015F	−19.236
N3021	1995al	−19.535
N3370	1994ae	−19.161
N3447	2012ht	−19.207
N3972	2011by	−19.103
N3982	1998aq	−19.507
N4038	2007sr	−19.058
N4424	2012cg	−19.534
N4536	1981B	−19.293
N4639	1990N	−19.113
N5584	2007af	−19.085
N5917	2005cf	−19.255
N7250	2013dy	−19.196
U9391	2003du	−19.449

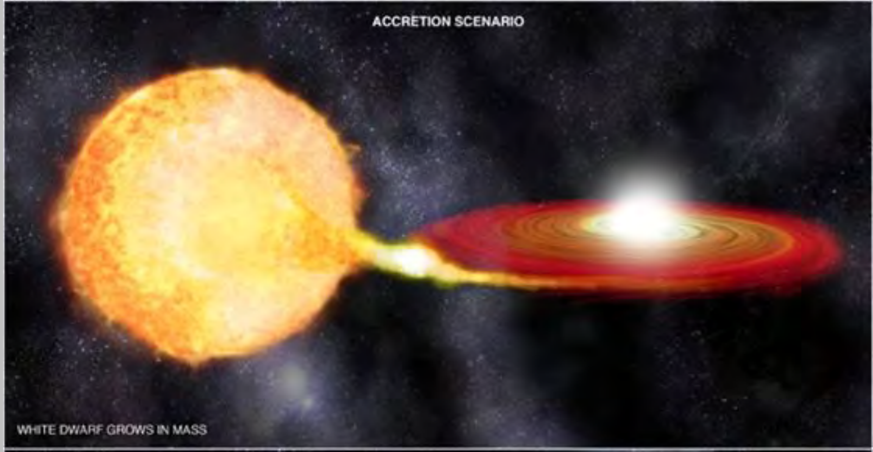
Are they "standard" enough?

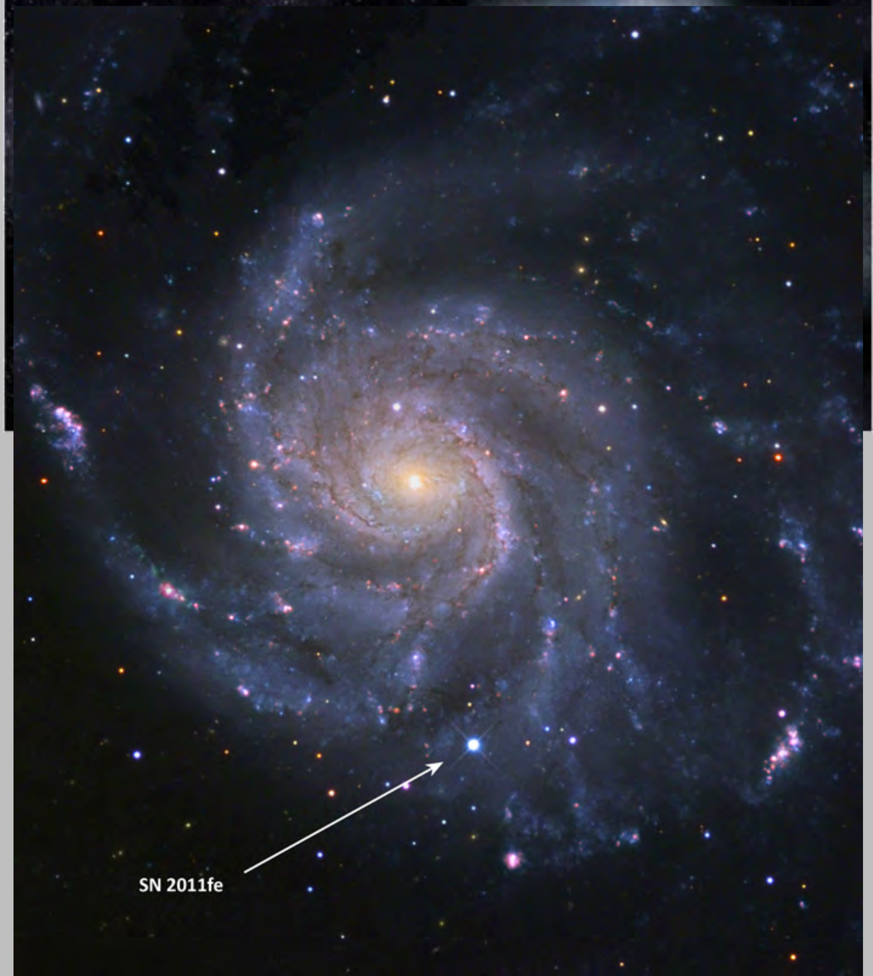
Calibration from Riess+16

$$\langle M_B \rangle = -19.26 \pm 0.16$$

BUT

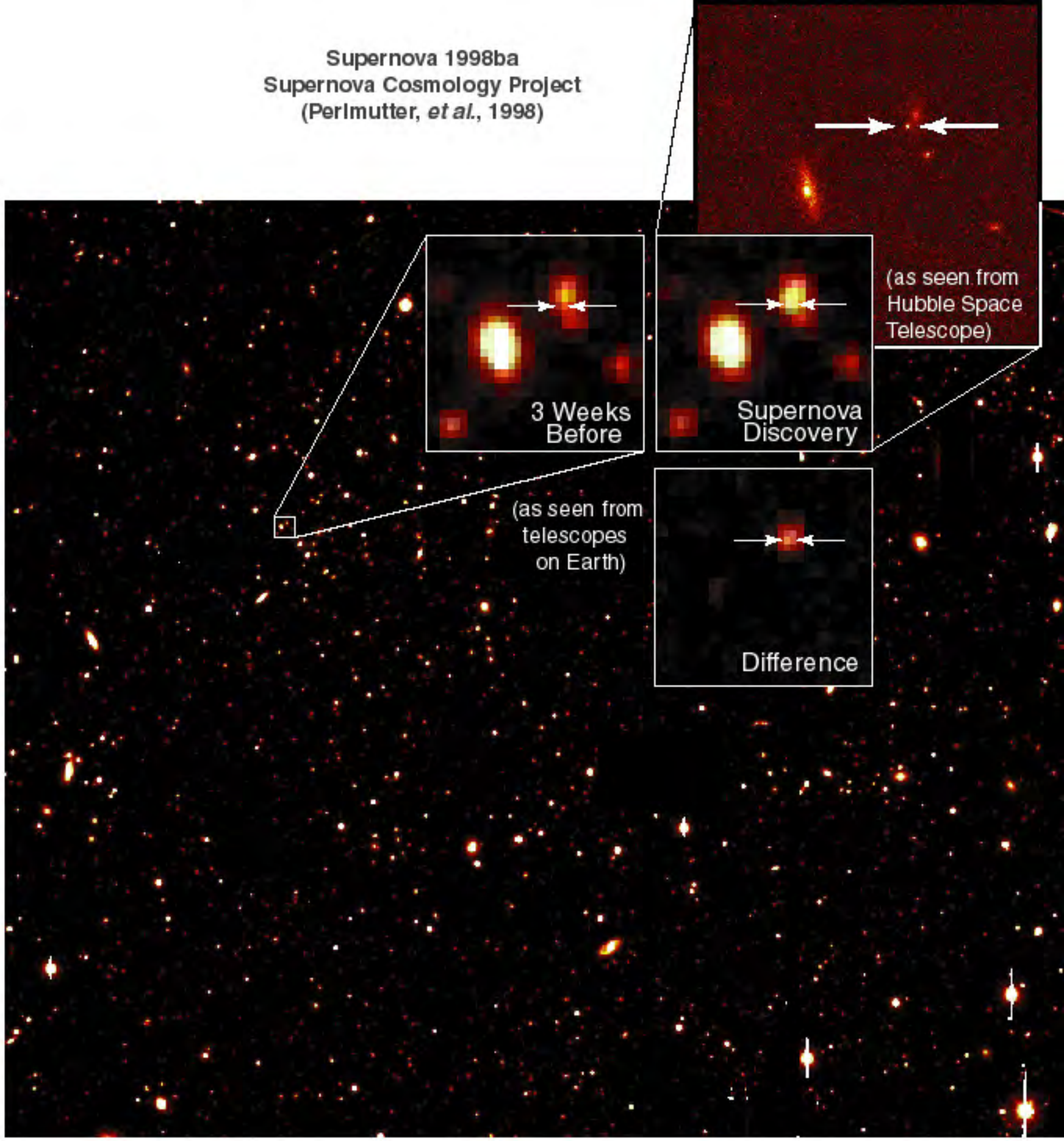
- Type Ia SNe are rare.
- You have to find them.
- You have to make sure they aren't a different type of SNe.
- And you have to hope Type Ia SNe at high redshift (in early universe) aren't different from the ones nearby!





Supernovae Cosmology Project

1. Take a deep, wide field image of a patch of sky, containing hundreds of galaxies.
2. Wait a few weeks, do it again. Look for differences: a possible supernova!
3. Take a spectrum of the supernova, make sure it actually is a Type Ia.
4. Take many images of the object over time to work out its light curve and derive its peak apparent magnitude.
5. Do this many times to build up the dataset.



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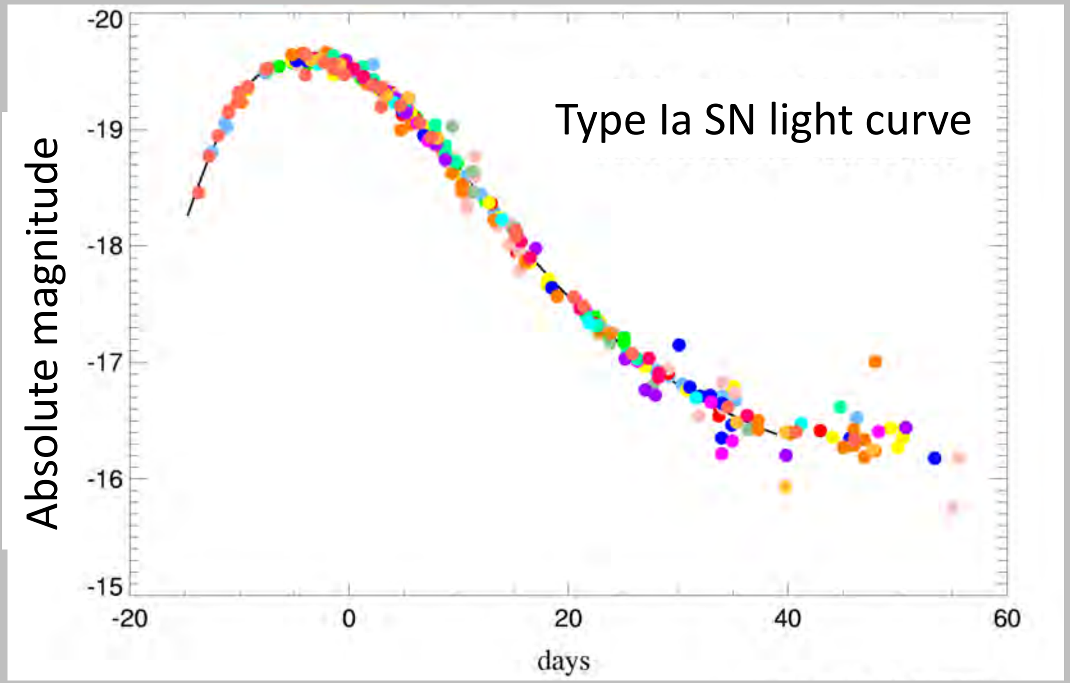
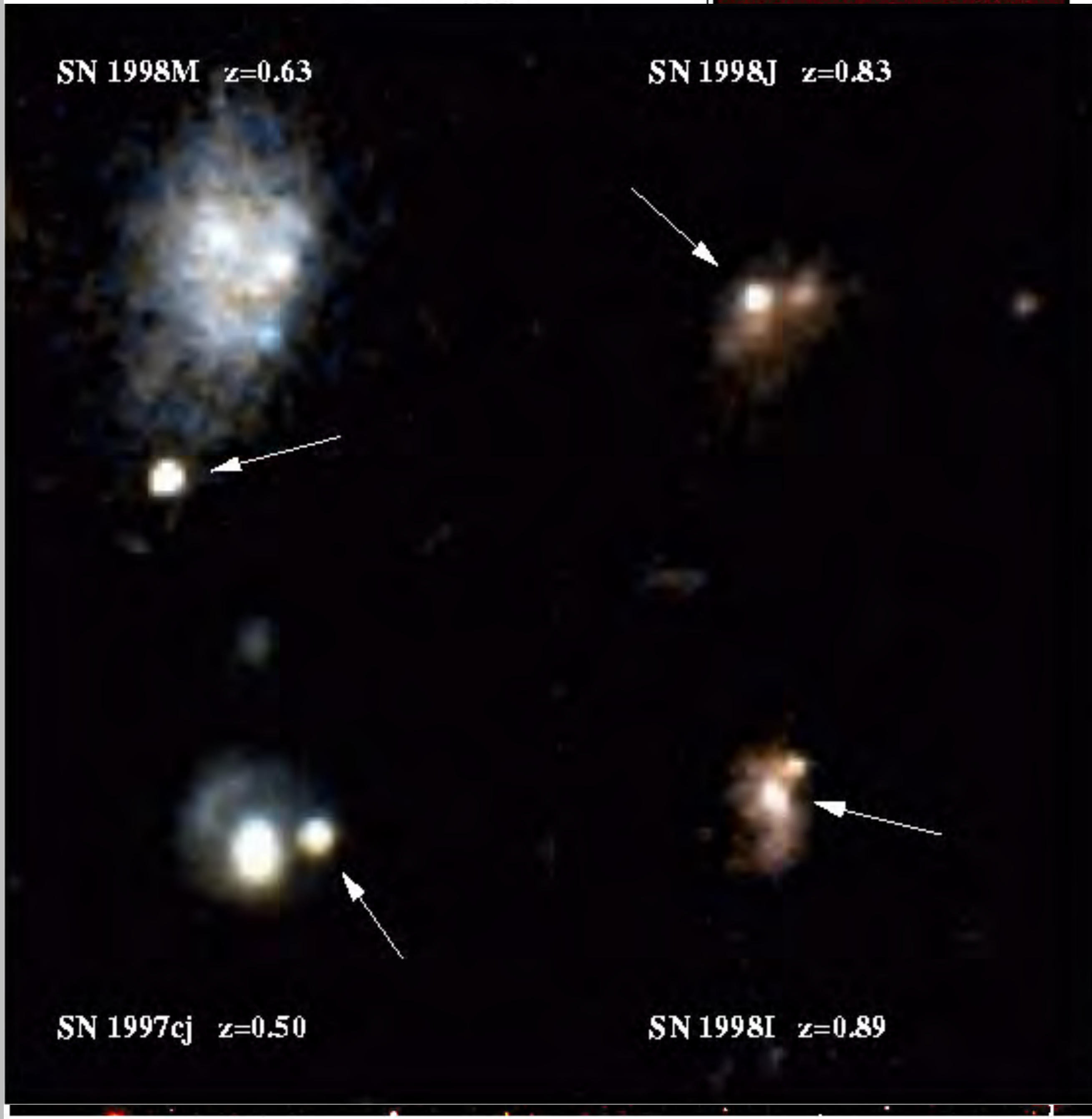


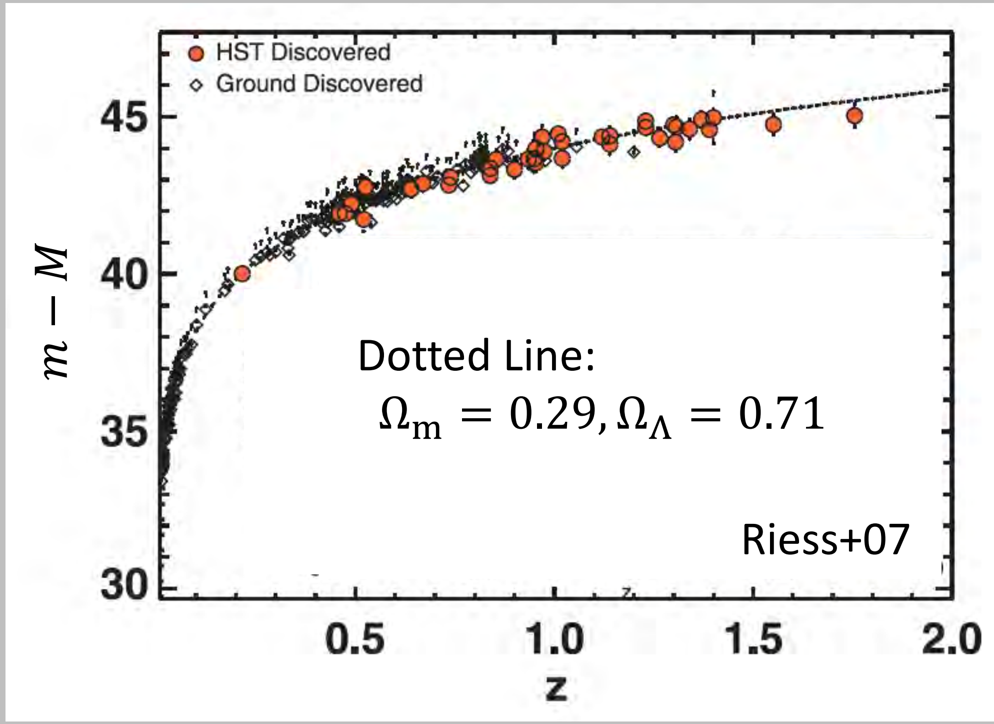


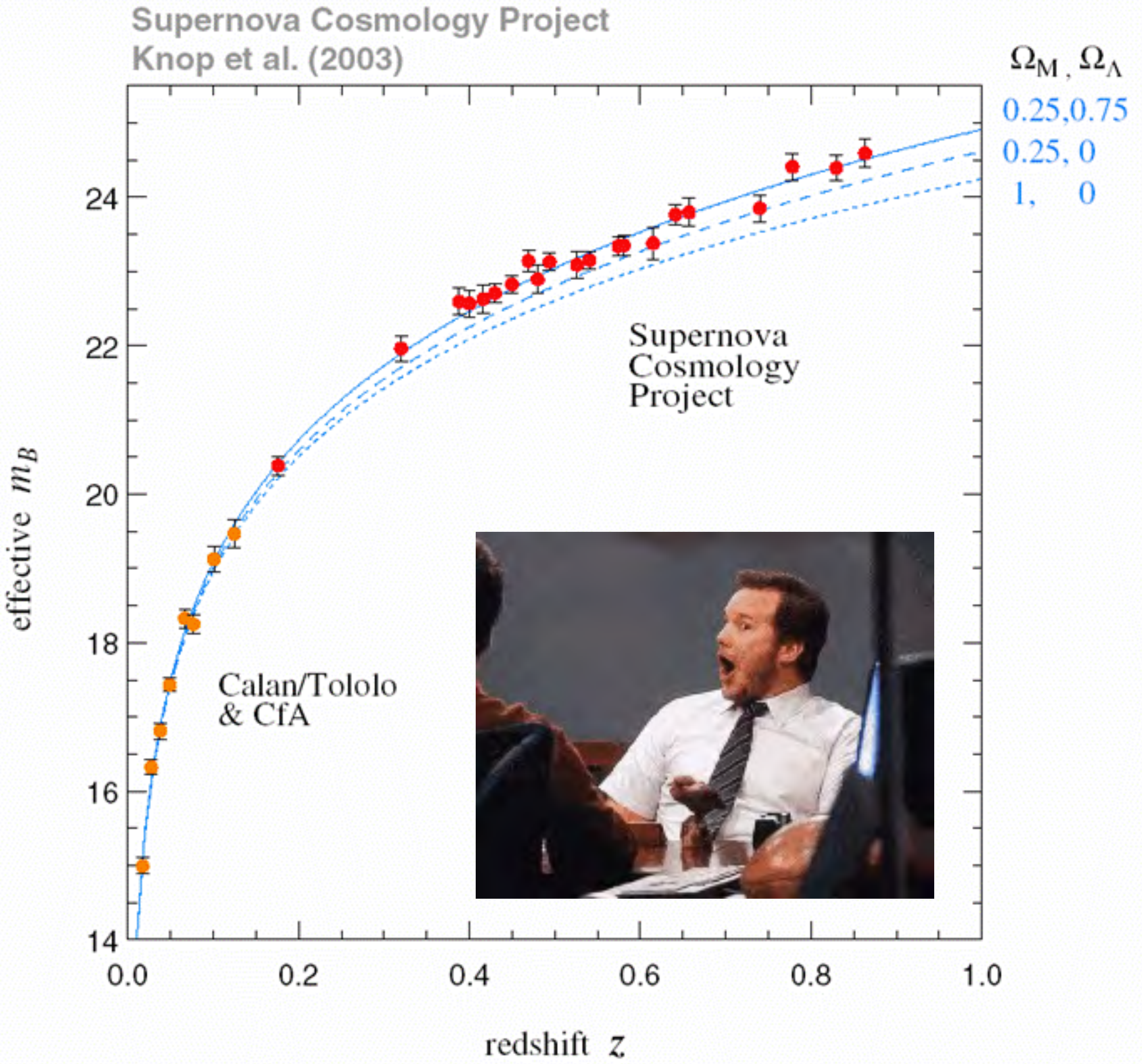
And the answer is…..

…not what people expected!

$$\Omega_{\mathrm{m}} = 0.25, \Omega_{\Lambda} = 0.75$$

The same result was obtained seperately and nearly simultaneously by two different research groups, and has been subsequently verified by several others.





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$$\Omega_\mathrm{m} \approx 0.2 - 0.5$$

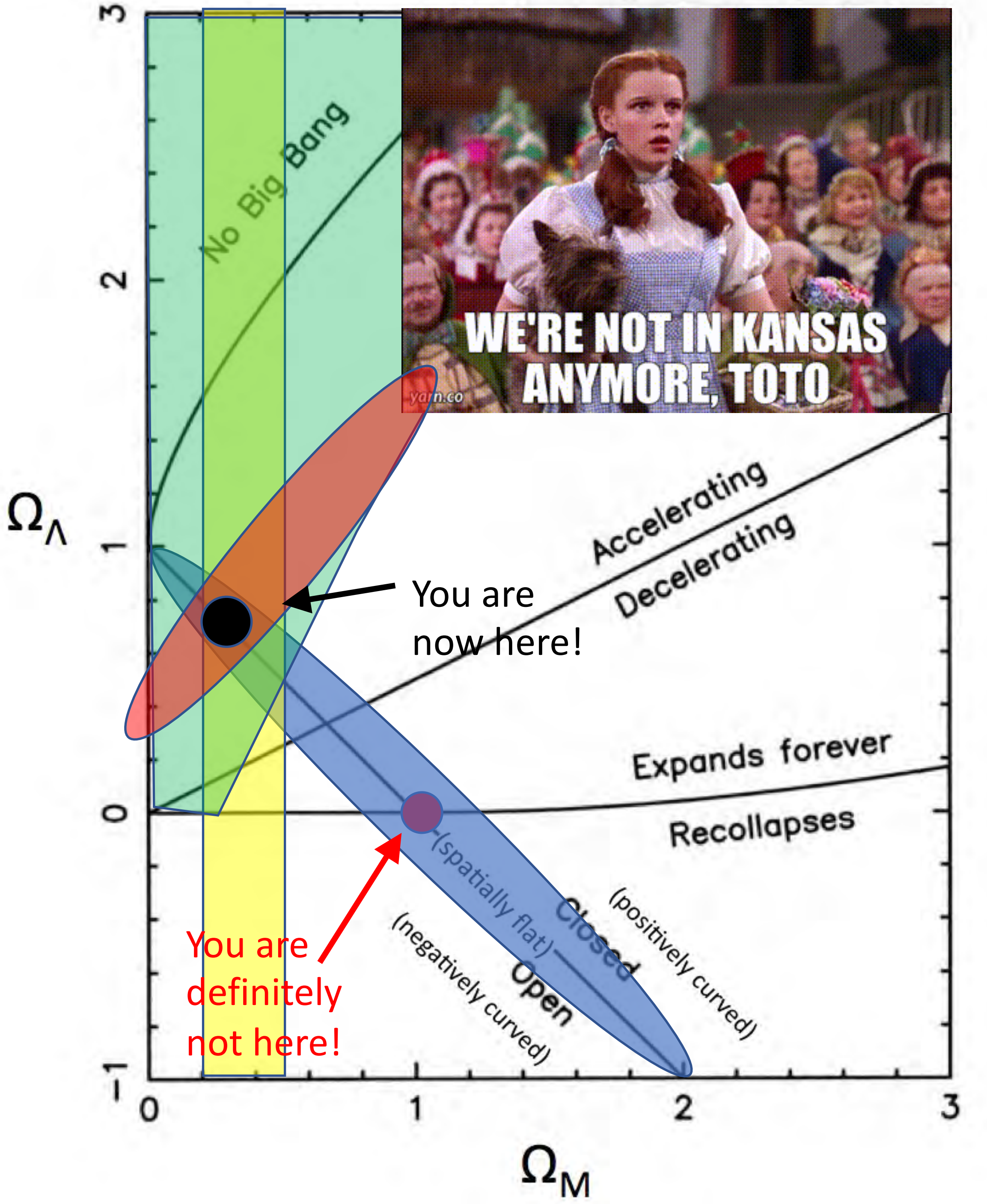
The fluctuations in the CMB continue to demand a flat universe

$$\Omega_\mathrm{m} + \Omega_\Lambda = 1$$

Supernovae cosmology shows acceleration:

$$\Omega_\Lambda - \Omega_\mathrm{m} \approx 0.4$$

Concordance cosmology: $\Omega_\mathrm{m} \approx 0.3, \ \Omega_\Lambda \approx 0.7$



The Cosmological Constant, Lambda, Dark Energy : time to take it seriously

Back to 1919: Einstein introduces the cosmological constant to keep the Universe static: But he threw it out once Hubble had demonstrated the Universe was not static. ***Now it's back.***

Dynamics Equation	Friedmann Equation
$\frac{\ddot{R}}{R} = -\frac{4\pi}{3} G\rho + \frac{1}{3}\Lambda c^2$	$\left(\frac{\dot{R}}{R}\right)^2 - \frac{8}{3}\pi G\rho - \frac{1}{3}\Lambda c^2 = -\frac{kc^2}{R^2}$
ρ decelerates the Universe Λ accelerates the Universe	ρ and Λ work together to set the shape of space.

The cosmological constant Λ acts like an energy (**"dark energy"**) providing an outwardly accelerating pressure, but working with matter to curve space.

(Remember Einstein: $E = mc^2$, so space responds to the matter-energy equivalency....)

But.... what is it?

Dark Energy: We really don't know what it is.

Simplest idea is that it is the **energy density of empty space**, perhaps due to virtual particles. As space expands, there is more space and so dark energy continues to grow in dominance compared to matter.

Virtual Particles from the Heisenberg Uncertainty Principle

Remember the Heisenberg Uncertainty Principle : $\Delta E \Delta t = \hbar$
And use $E = mc^2$ to rewrite it as $\Delta m \Delta t = \hbar / c^2$

On small enough scales, the amount of mass or energy in a vacuum is uncertain. Particles can pop in and out of existence, being created and then almost instantly annihilated, on length- and time-scales that are unobservable.

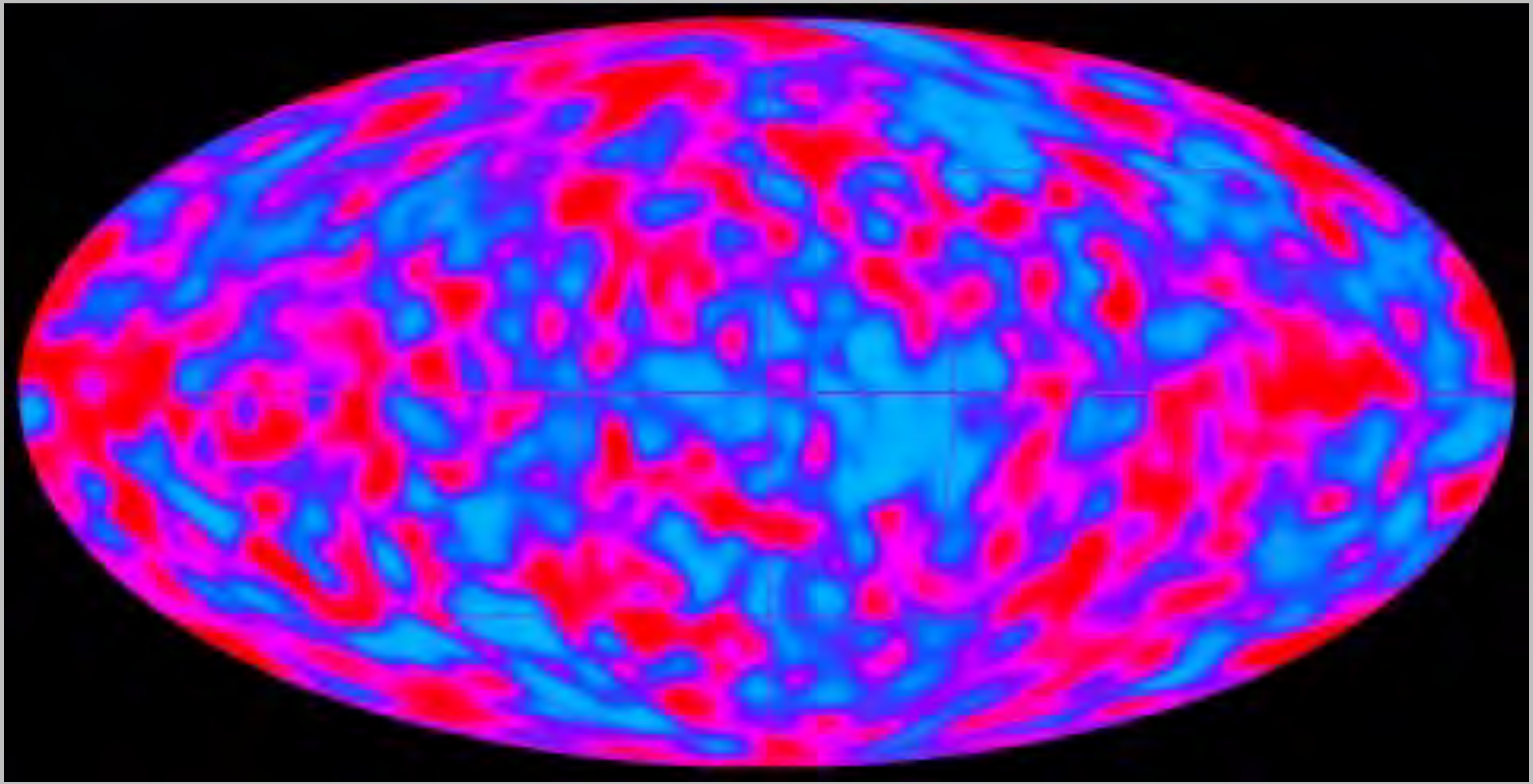
- Theoretical estimate of energy density due to "virtual particles": $\approx\approx\approx 10^{111}$ J m^{-3}
- Observational measurement of the energy density associated with $\Lambda := 6{\times}10^{-10}$ J m^{-3}

Only off by 120 orders of magnitude!

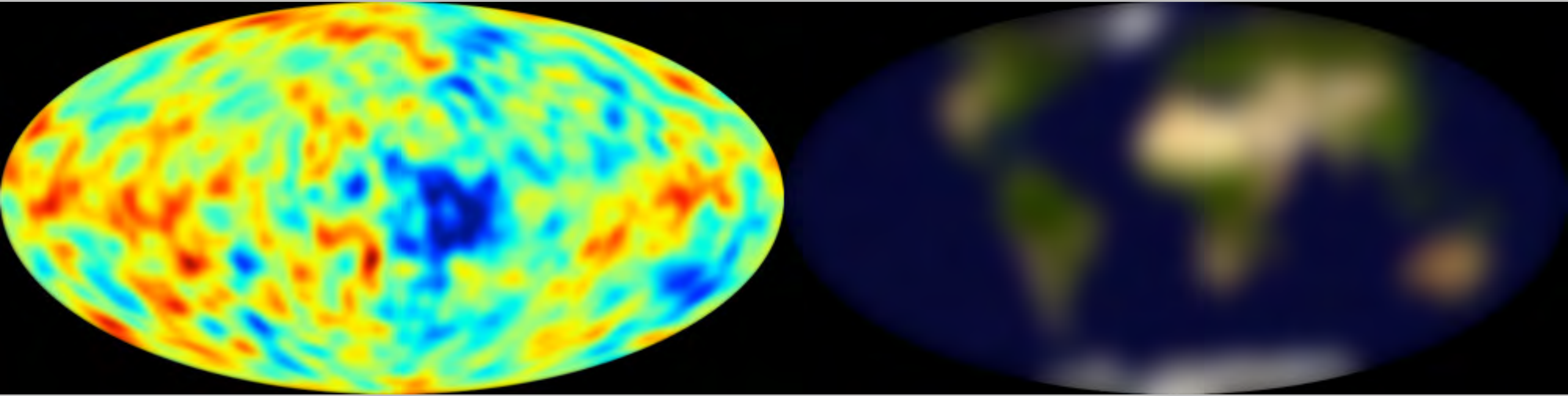
More work is needed.

Meanwhile, better data began coming in for the cosmic microwave background (CMB)

COBE all-sky microwave map (1992)

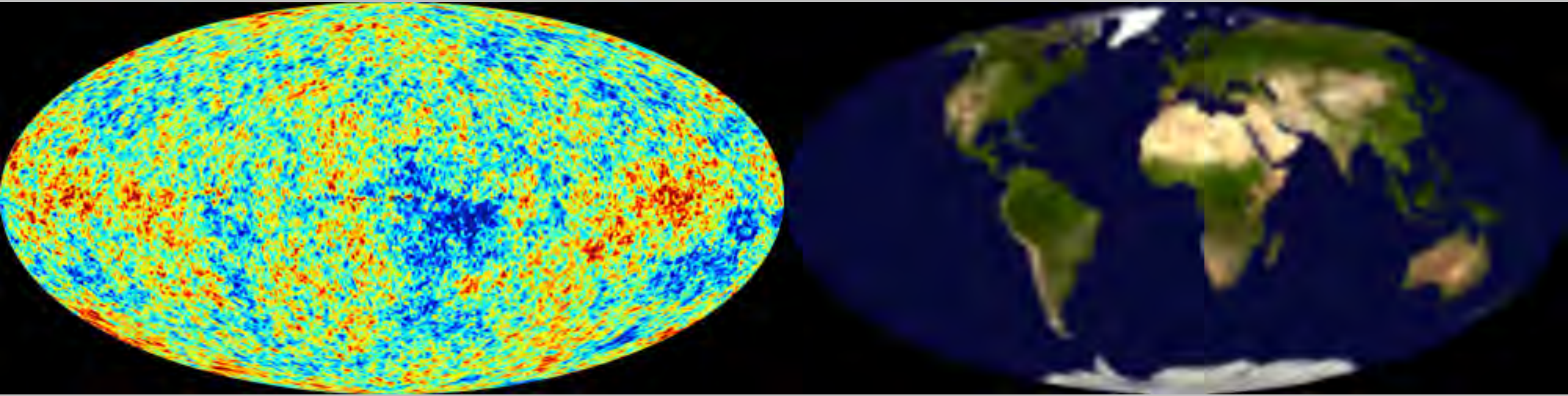


COBE all-sky CMB map (1992)



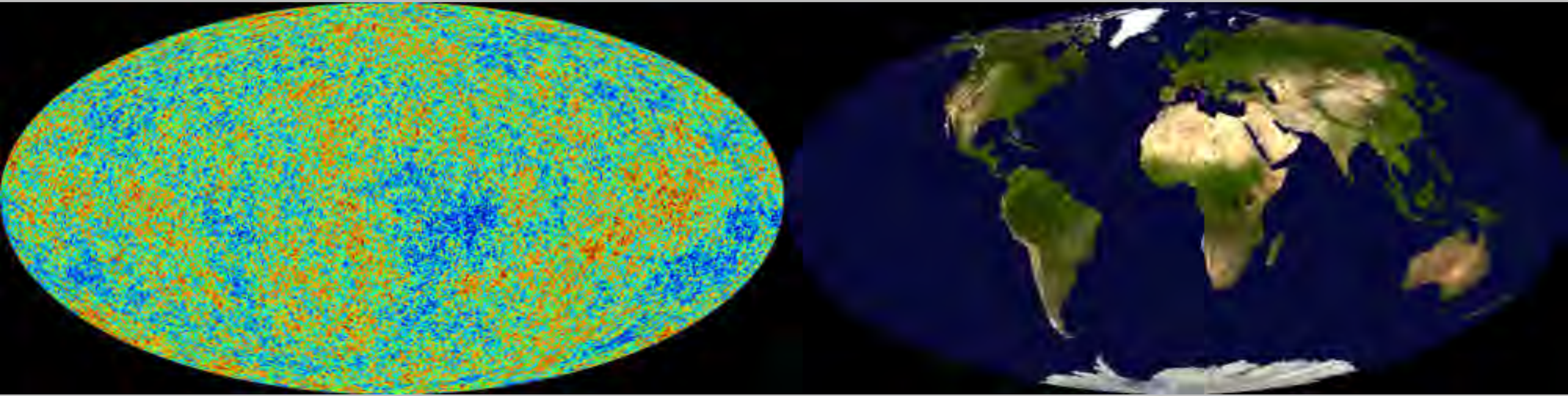
Graphics from New Scientist

WMAP all-sky CMB map (2003)



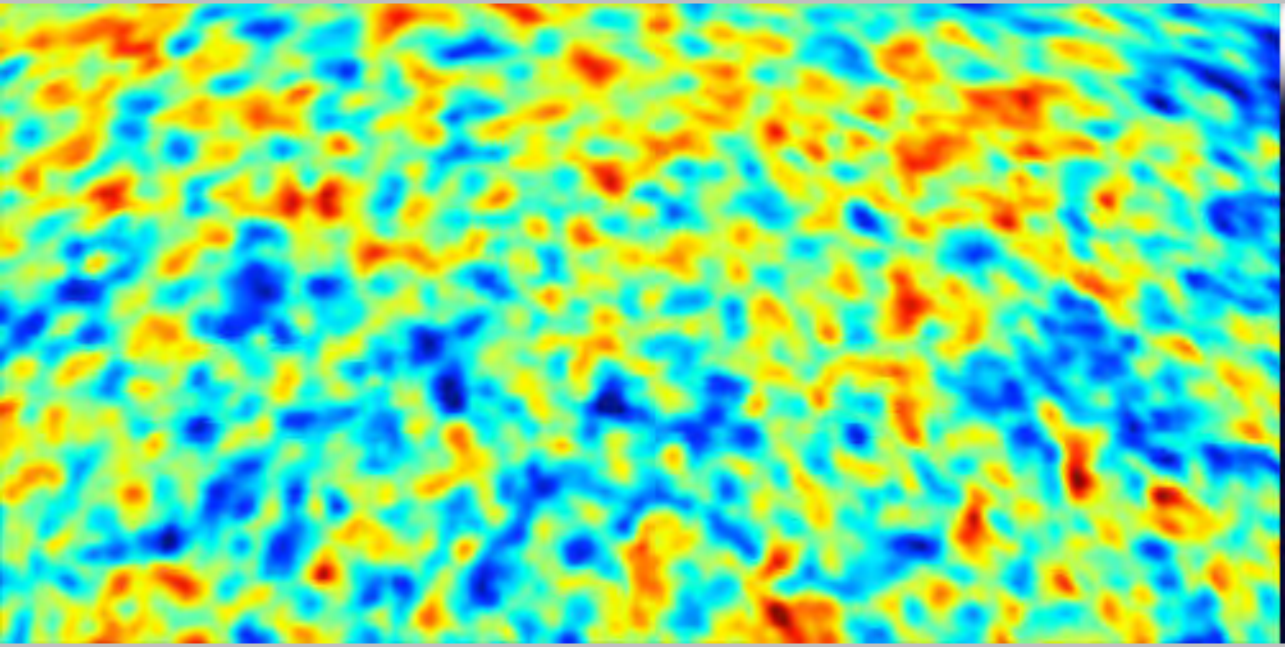
Graphics from New Scientist

Planck all-sky CMB map (2013)



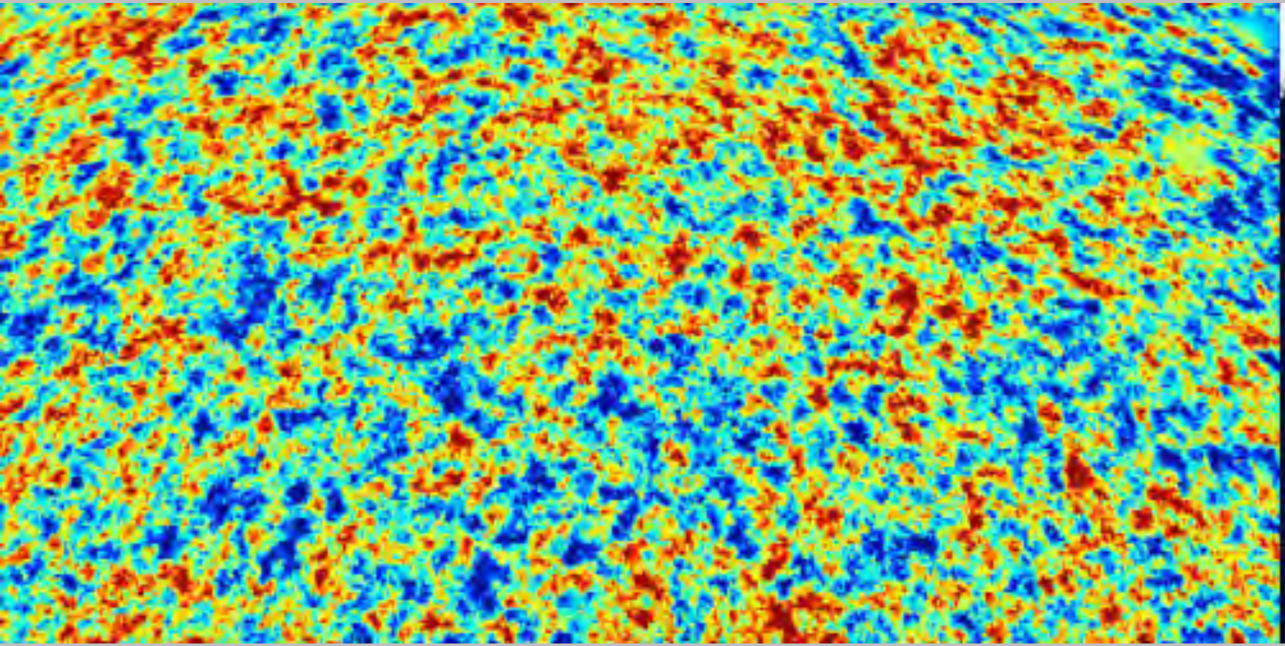
Graphics from [New Scientist]

WMAP zoomed in (2003)



Graphics from [New Scientist]

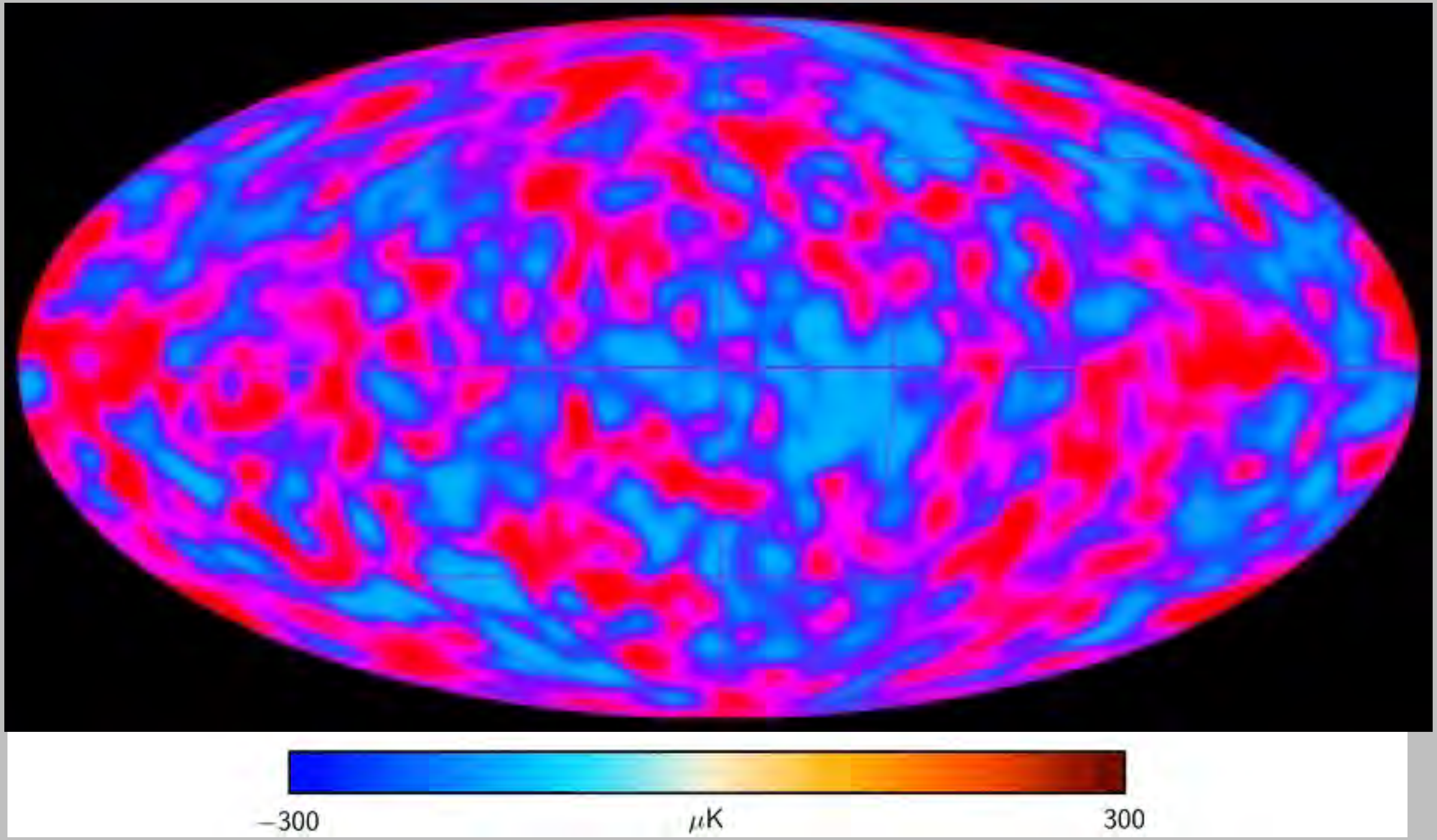
Planck zoomed in (2013)



Graphics from New Scientist

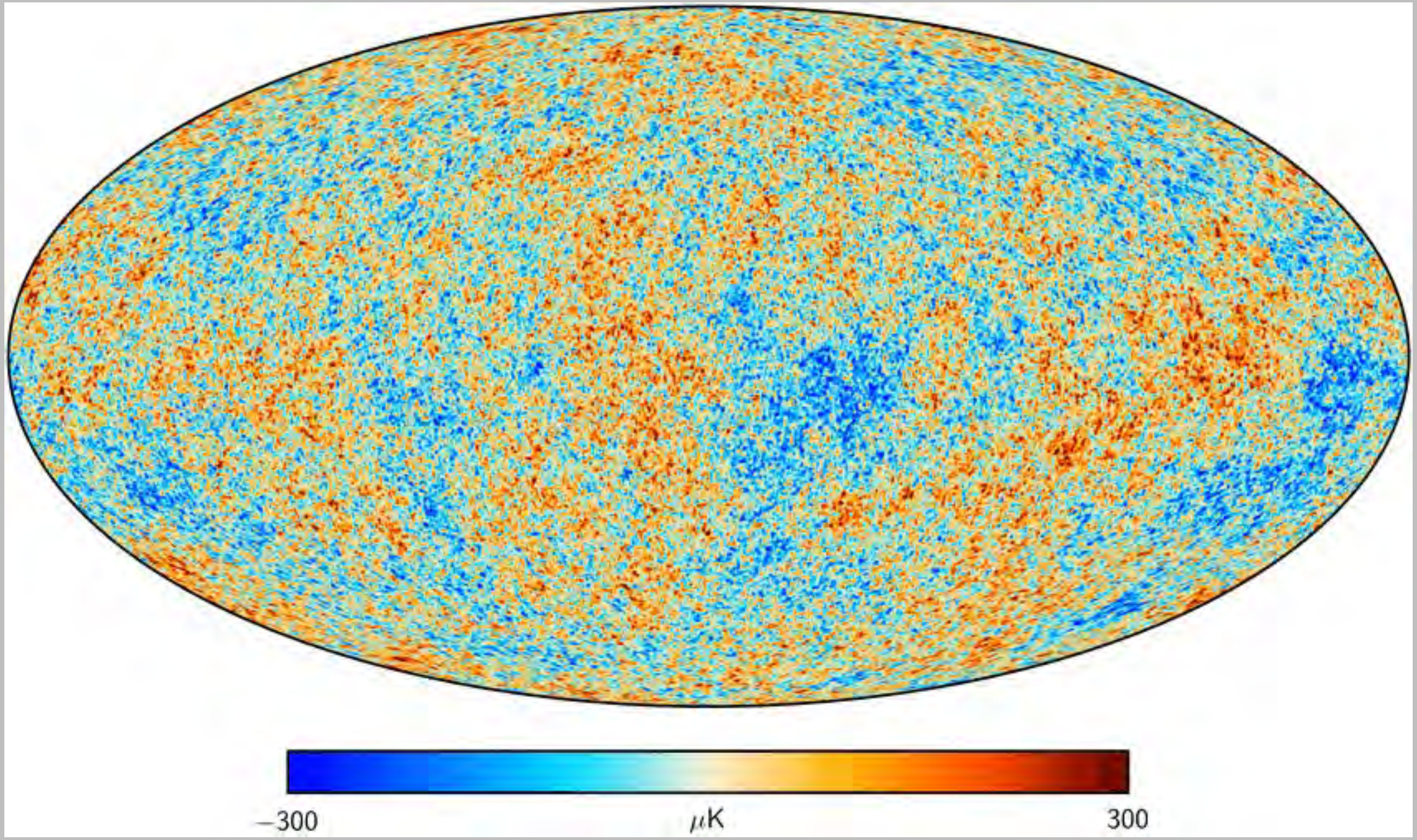
Satellite cosmic microwave background measurements: WMAP (mid 2000s) and Planck (mid 2010s)

COBE all-sky microwave map (1992)



Satellite cosmic microwave background measurements: WMAP (mid 2000s) and Planck (mid 2010s)

Planck all-sky microwave map (2018)



The structure of the CMB on smaller scales is sensitive to other cosmological parameters ($H_0, \Omega_m, \Omega_\Lambda, \Omega_{baryon}, \ldots$).

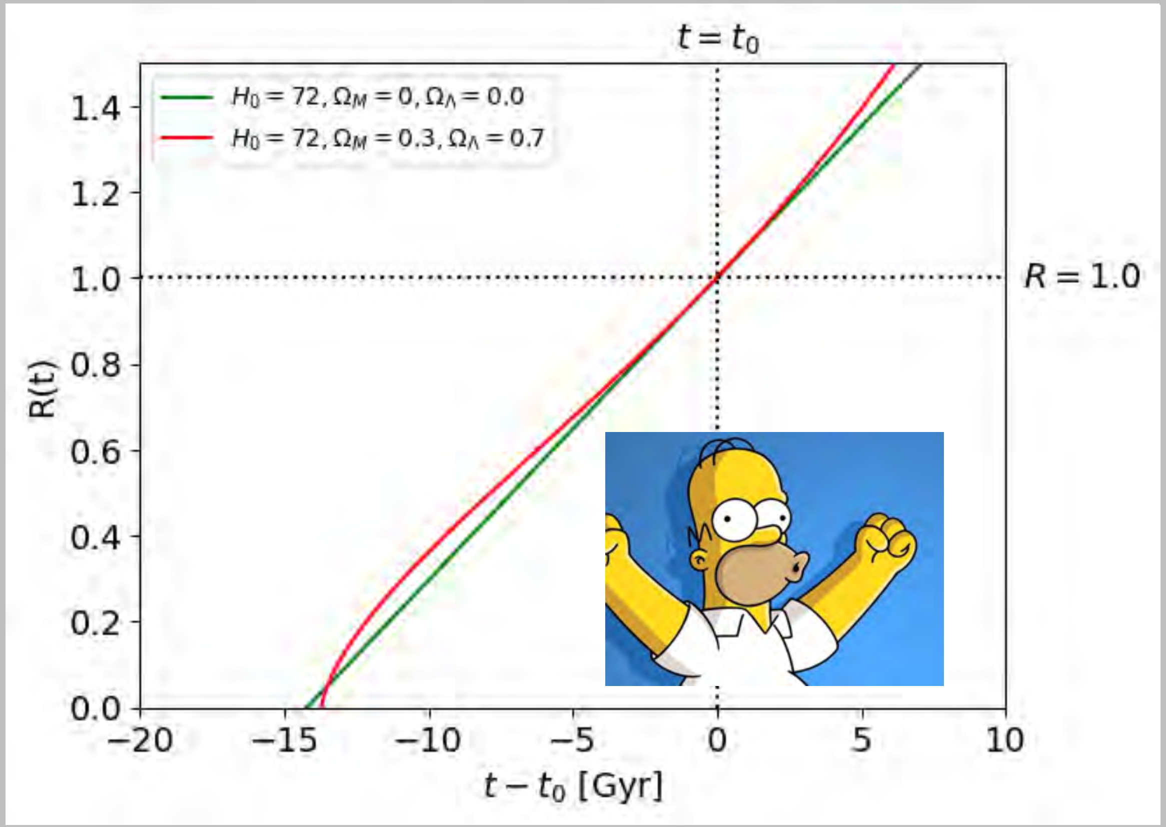
(More on this in ASTR 328!)

The most recent estimates (Planck 2018) give:

Parameter	Value
H_0	67.7 km/s/Mpc
$\Omega_{m,0}$	0.31
$\Omega_{\Lambda,0}$	0.69
$\Omega_{baryon,0}$	0.049

The (Basic) Cosmological Parameters: Best estimates

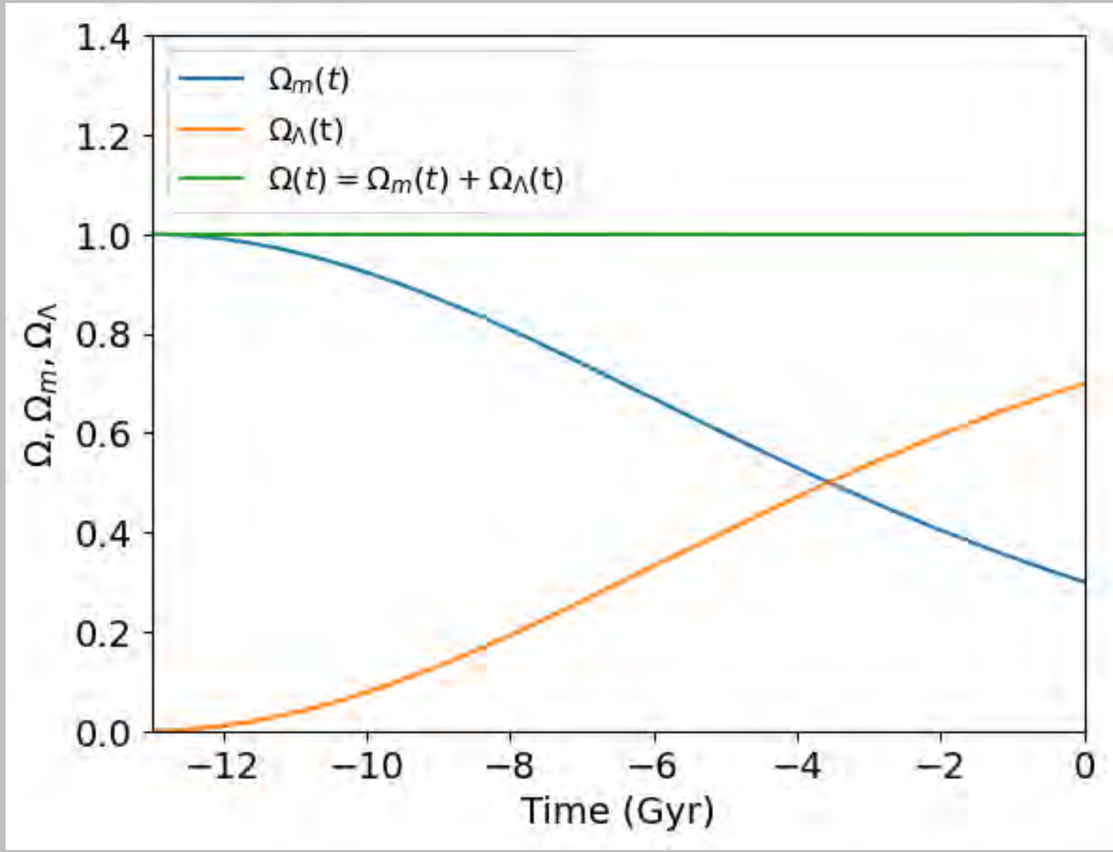
$$H_0 \approx 72\, km/s\,/Mpc$$
$$\left.\begin{array}{r}\Omega_{m,0} \approx 0.3 \\ \Omega_{\Lambda,0} \approx 0.7\end{array}\right\} \Omega_0 = 1$$
$$\Rightarrow t_0 = 13.6 \text{ Gyr}$$



Remember, the parameters change with time as the Universe expandsΩ_m starts at 1 ("matter dominated"), but drops over as the Universe expands and the density drops.

Ω_Λ starts at 0 since matter dominates at early times, but rises over as the Universe expands and the density drops.

Total $\Omega = 1$ always: a spatially flat universe stays flat.



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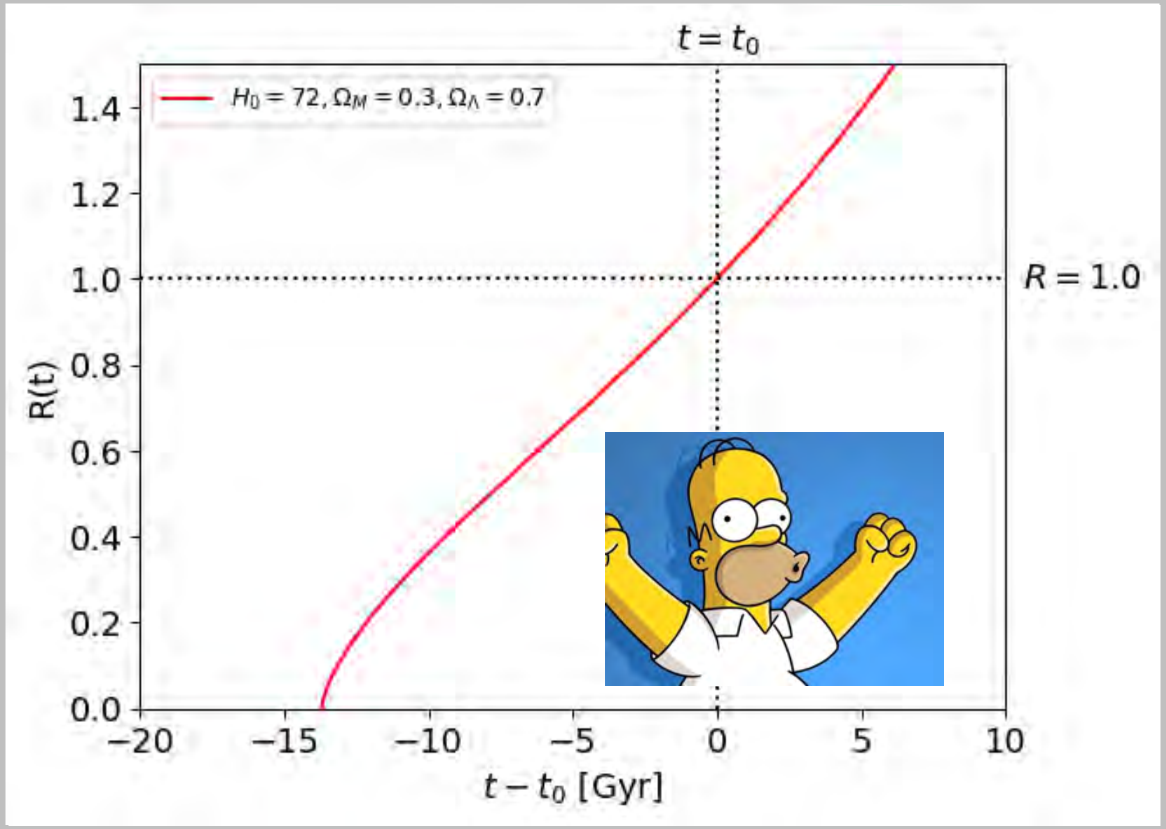

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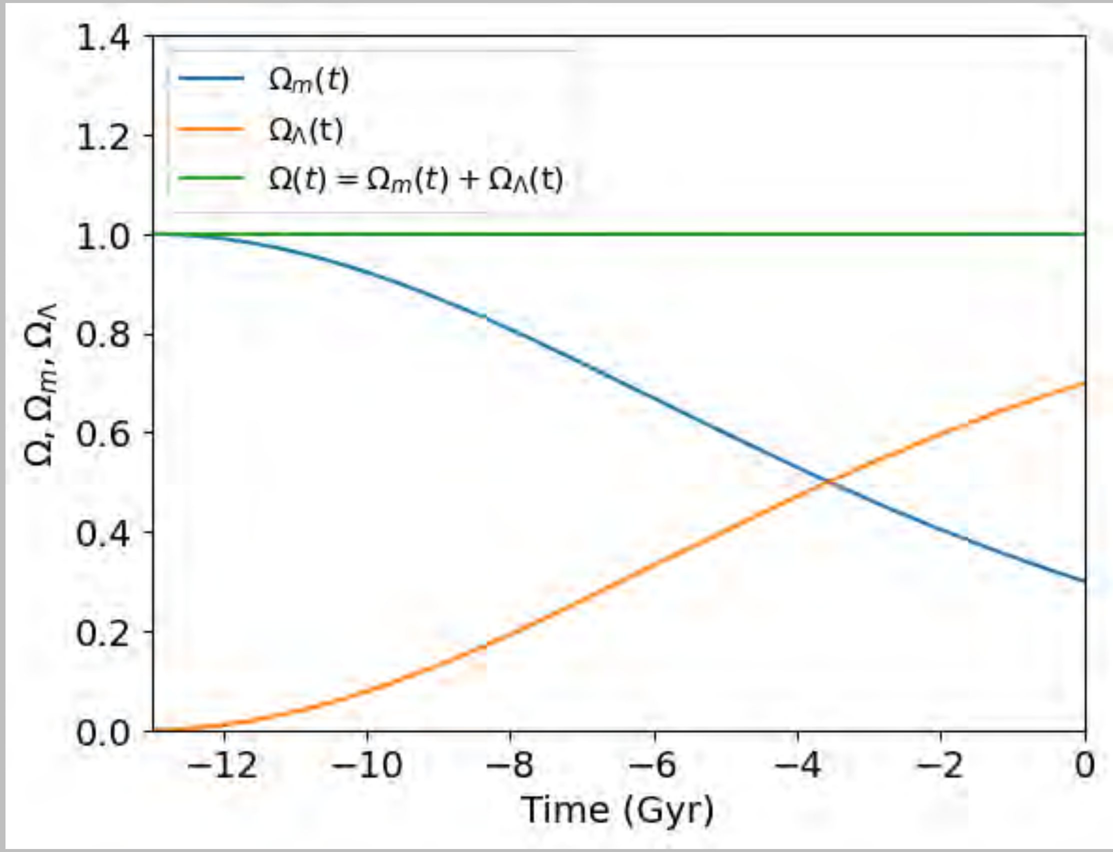
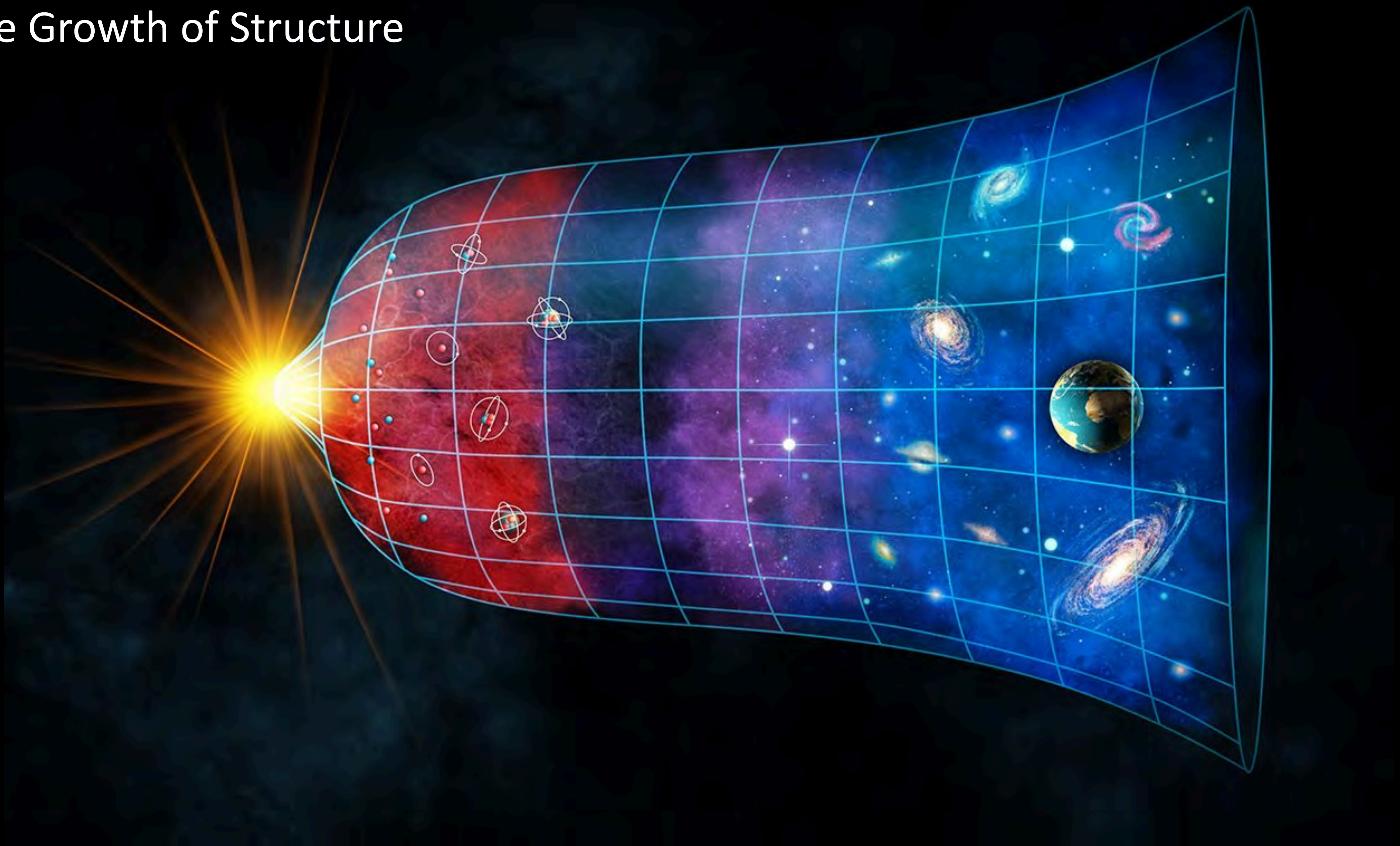


The Growth of Structure



The Early Universe: after inflation

Universe is small, dense, very hot, and filled with elementary particles. At these energies we get **pair production**: particles and antiparticles being destroyed and reforming.

For example, electrons (e^-), positrons (e^+), and gamma rays (γ):

$$e^+ + e^- \Leftrightarrow \gamma + \gamma$$

annihilation ⇒

⇐ creation

and similiarly for other particles.

As the Universe expands, gamma rays are being redshifted and losing energy, and after a few micro-seconds, they don't have enough energy to create particles. But particles can still be destroyed by annihilating with their anti-particles, so in a flash we lose all our matter, converting it via annihilations into high energy radiation (destined to be redshifted to form the CMB).

OK, we don't lose all the matter, some must have survived. To match the total matter the amount of mass in the Universe today with the number of CMB photons we see, for every 10^9 anti-particles there must have been 10^9+1 particles: the ***matter-antimatter asymmetry***.

For every 10^9 annihilations, one unpaired particle survives. Ask the physicists about that!

Big Bang Nucleosynthesis: Cooking up elements

Now we have the building blocks for the elements: protons (p), neutrons (n), electrons (e^-). The temperature and density is high, but dropping fast as the Universe expands. If we act quick, we might be able to drive nuclear fusion.

Step 1: Assemble your ingredients

At high energies ($t < 1$ second), interactions are transforming particles back and forth:

$$n \Leftrightarrow p + e^- + \bar{\nu}_e$$

neutrino

The particles are in thermal equilibrium, so that the proton-neutron ratio is given by the **Boltzmann equation**:

$$\frac{N_n}{N_p} = e^{-\Delta E/kT} = e^{-(m_p - m_n)c^2/kT}$$

at $t \approx 1$ second, $\mathrm{T} \approx 10^{10}$ K, so the ratio is $N_n/N_p = 0.223$. Below that temperature, those reactions stop and the ratio is frozen in.

But it is too hot for sustained fusion, so that ratio is maintained: for every 1000 protons, there are 223 neutrons.

Big Bang Nucleosynthesis: Cooking up elements

Now we have the building blocks for the elements: protons (p), neutrons (n), electrons (e^{-}). The temperature and density is high, but dropping fast as the Universe expands. If we act quick, we might be able to drive nuclear fusion.

Step 2: Make a mess and spill your ingredients (1000 protons, 223 neutrons).

So for every 1000 protons, there are 223 neutrons, but we are waiting for things to cool.

But free neutrons are unstable and undergo beta decay:

$$n \Rightarrow p + e^{-} + \bar{\nu}_e$$

with a half-life of about 10 minutes. So for every 10 minutes you wait, you lose half of the neutrons you had, converting them to protons.

To begin fusion, you need the temperature to drop from $\mathrm{T} \approx 10^{10}$ K to $\mathrm{T} \approx 10^{9}$ K, and that takes about 4 minutes, at which point the neutron-to-proton ratio has dropped from $N_n/N_p = 0.223$ to $N_n/N_p = 0.164$.

So by the time you start fusion, your original mix has turned into 1051 protons, there are now 172 neutrons.

Big Bang Nucleosynthesis: Cooking up elements

Now we have the building blocks for the elements: protons (p), neutrons (n), electrons (e^-). The temperature and density is high, but dropping fast as the Universe expands. If we act quick, we might be able to drive nuclear fusion.

Step 3: Start cooking – nuclear fusion! (1051 protons, 172 neutrons)

At 10^9 K, the oven is ready, and we begin fusion.

1. Protons and neutrons fuse to form deuterium and release a gamma ray:

$$p + n \Leftrightarrow {}^2_1H + \gamma$$

2. Deuterium fuses to form tritium and a proton:

$${}^2_1H + {}^2_1H \Leftrightarrow {}^3_1H + p$$

3. Tritium and deuterium fuse to form helium and a proton:

$${}^3_1H + {}^2_1H \Leftrightarrow {}^4_2He + p$$

Net result: convert 4 protons into 1 helium nucleus.

(But note: the reaction chain is different from how the Sun and stars do it!)

Big Bang Nucleosynthesis: Cooking up elements

Now we have the building blocks for the elements: protons (p), neutrons (n), electrons (e^-). The temperature and density is high, but dropping fast as the Universe expands. If we act quick, we might be able to drive nuclear fusion.

Step 4: Taste the dish – how did it turn out?

If the fusion process is 100% efficient, at the end of this process we have made all the Helium we possibly could have. How much Helium is that?

We started fusion with 1051 protons and 172 neutrons. Since a Helium nucleus has 2 protons and 2 neutrons, we can make a total of 172/2 = 86 Helium nuclei. They would use up 172 protons as well, leaving you with 1051 − 172 = 879 protons.

This means the fraction of Helium by mass is given by:

$$Y = \frac{4\times86}{(1\times879 + 4\times86)} = 0.28$$

Which is pretty close to the primordial helium abundance (measured in low metallicity stars) of $Y \approx 0.23 - 0.24$.

⇒ Most of the Helium in the Universe was made during the Big Bang, and not inside stars!

BBN constraints on baryonic matter density (Ω_b).

The efficiency of BBN depends on the density of baryonic (normal) matter in the Universe:

Higher density ⇒ more collisions ⇒ more efficient fusion
⇒ *More helium, fewer leftovers* (e.g., deuterium).

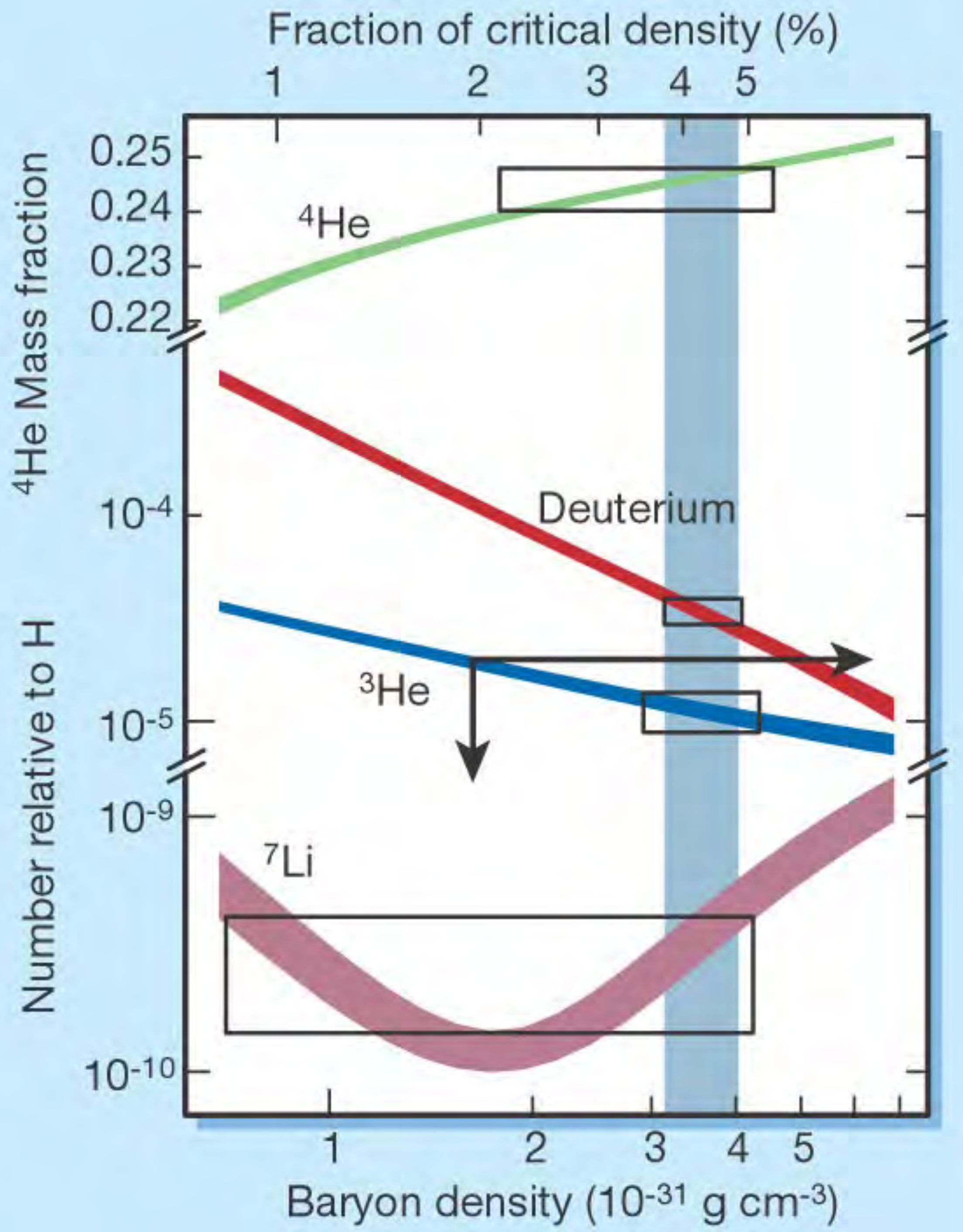
Lower density ⇒ fewer collisions ⇒ less efficient fusion
⇒ *Less helium, more leftovers*.

The primordial abundances of helium, deuterium, and a few other of the “light elements” depend on the baryon density of the universe

Comparing to observed values shows that

$$\Omega_b \approx 0.04$$

And since $\Omega_m \approx 0.25 - 0.3 \gg \Omega_b$, **dark matter cannot be made of normal baryonic matter.**



The transition from radiation dominated era to matter dominated era

After BBN, the universe was filled with matter and radiation. The radiation energy density dominates that of the mass, and we are in the **radiation dominated era**. But radiation density drops faster than mass density as the expansion continues.

Mass energy density:

$$U_{matter} = \frac{Mass \times c^2}{Volume}$$

$$U_{matter} \sim R^{-3}$$

Radiation energy density:

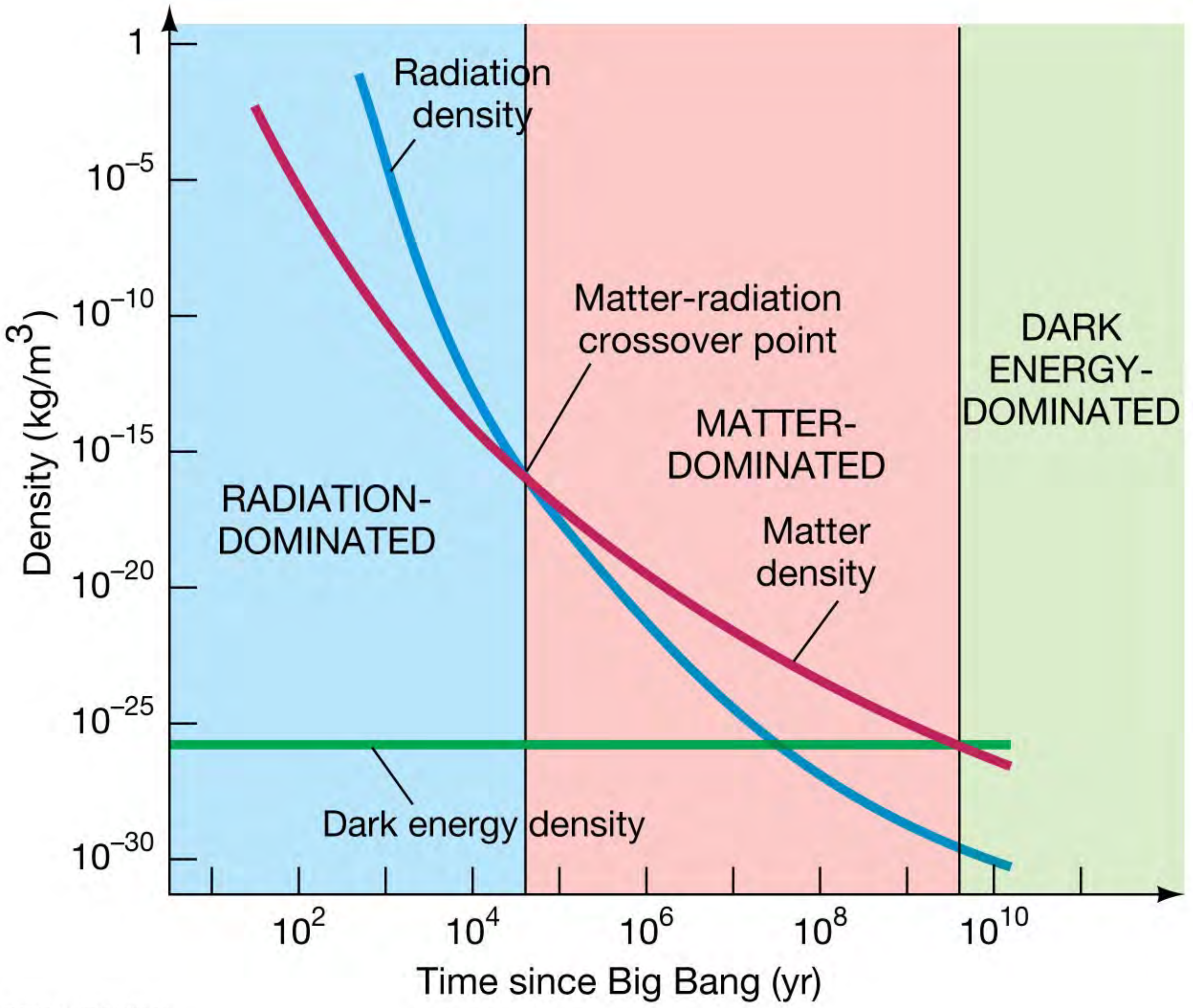
$$U_{rad} = \frac{N_{photon} \times E_{photon}}{Volume}$$

Photon energies drop as they get redshifted!

$$U_{rad} \sim \frac{R^{-1}}{R^3} \sim R^{-4}$$

At $t \approx 55{,}000$ years ($T \approx 9000$ K), radiation energy density drops below mass energy density, and we enter the **matter-dominated era**.

Time for gravity to do its thing!



Matter in the early universe

Two forms of matter:
- **Baryonic** (normal matter: protons, neutrons; including electrons), 10% – 15% of total mass
- **Non-baryonic dark matter** (????), 85% − 90% of total mass

Focus now on baryonic matter.

Before recombination ($t \lesssim 350,000$ years):

Photons scatter off of free electrons (Thompson scattering), providing photon pressure which prevents baryons from falling into gravitational potential wells. They are "suspended".

After recombination ($t > 350,000$ years)

Free electrons combine with free protons to form hydrogen atoms, no more Thompson scattering, no more photon pressure. Baryons can start falling into potential wells.

Matter in the early universe

Two forms of matter:

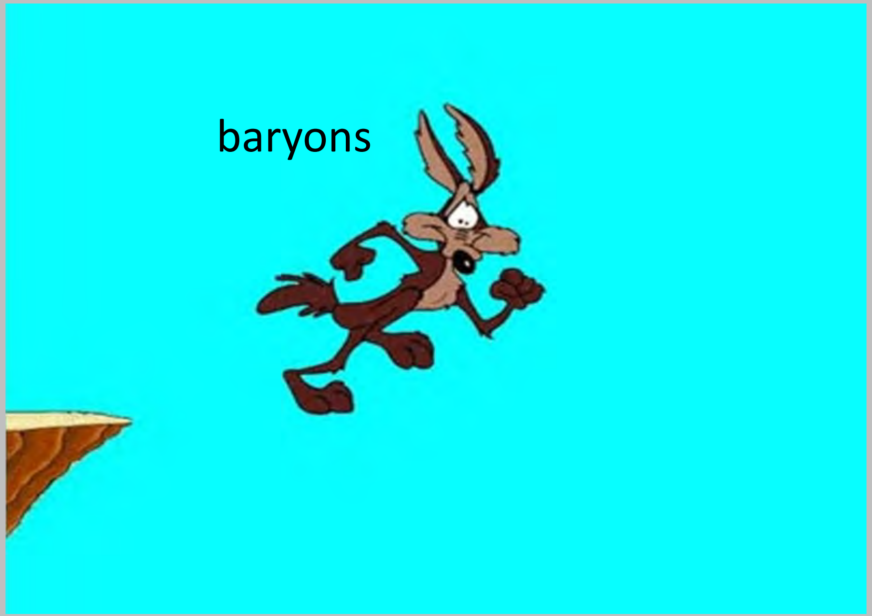
- **Baryonic** (normal matter: protons, neutrons; including electrons), 10% – 15% of total mass
- **Non-baryonic dark matter** (????), 85% − 90% of total mass

In the early universe ($t \lesssim 350{,}000$ years) the Universe is too hot for bound hydrogen to form, so all the baryonic matter is ionized: free protons and electrons, with some helium nuclei and other leftovers from BBN. All that ionized baryonic matter is mixed with photons and dark matter.

Remember: photons and electrons easily scatter off one another (Thompson scattering), which is why the early universe is opaque: Light cannot free stream.

But that also means that photon pressure keeps the baryonic matter from gravitationally contracting.

Imagine an overdense lump of the universe at this time. The excess gravity is wanting to pull more mass inwards: gravitational contraction. But photon pressure keeps the electrons keep the from falling inwards: they stay "suspended" (along with the protons, which are electrostatically coupled to the electrons).



Recombination

At $t \approx 350{,}000$ years (redshift $z \approx 1000$), the temperature drops below $\mathrm{T} \approx 3{,}000$ K which is cool enough for protons and electrons to combine to form bound hydrogen atoms.

No more free electrons, no Thompson scattering of photons. The universe becomes transparent, the photon pressure goes away, and suddenly baryonic matter can start to collapse under gravity.

The Growth of structure

Because matter and radiation are coupled before recombination, the temperature fluctuations in the cosmic microwave background are related to the baryonic density fluctuations at recombination: at $z \approx 1000$, $\Delta T/T \approx \Delta\rho/\rho \approx 10^{-5}$.

Hubble (and now JWST) also detect lots of galaxies forming by $z \approx 5$. A galaxy is a very strong overdensity: $\Delta\rho/\rho \approx 10^{5}$.

This is orders of magnitude of growth in density in less than a billion years. *Can gravity work that fast?*

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Using the Friedman Equation to study structure formation

Remember, at early times the density is high enough that matter dominates over lambda. And also, observations told us that universe is spatially flat and stays flat. So to study the universe at these times, we can use the Friedman equation for a flat ($k = 0$) matter dominated ($\Lambda = 0$) universe.

$$\left(\frac{\dot{R}}{R}\right)^2 - \frac{8}{3}\pi G\rho - \frac{1}{3}\Lambda c^2 = -\frac{kc^2}{R^2}$$

$$H^2 - \frac{8}{3}\pi G\rho = 0$$

Hubble **parameter** – changes with time. Not Hubble constant (H_0)

Evolution of density fluctuations

To describe the Universe as a whole, start with the Friedman equation for a flat ($k = 0$) matter dominated ($\Lambda = 0$) universe.

$$H^2 - \frac{8}{3}\pi G\bar{\rho} = 0$$

average density of universe

Now consider a small piece of the universe (“a bubble”) which has a higher-than-average density: an overdensity. In that region, space is not flat, because it has more matter. So it gets its own Friedmann equation:

$$H^2 - \frac{8}{3}\pi G\rho' = -\frac{k}{R^2}$$

excess density of bubble

Now subtract the first equation from the second to get

$$-\frac{8}{3}\pi G(\rho' - \bar{\rho}) = -\frac{k}{R^2}$$

or, collecting terms:

$$(\rho' - \bar{\rho}) = -\frac{3k}{8\pi G R^2}$$

Evolution of density fluctuations (continued)

excesss density of bubble

average density of universe

Now lets define a quantity δ which is the **fractional overdensity** of the bubble. $\delta = 0$ is average density (i.e., no overdensity), $\delta = -1$ is absolute emptiness (a strong underdensity), while $\delta \gg 1$ is a strong overdensity.

$$\delta \equiv \left(\frac{\rho' - \bar{\rho}}{\bar{\rho}}\right)$$

Putting that into our equation for the difference between ρ' and $\bar{\rho}$

$$(\rho' - \bar{\rho}) = -\frac{3k}{8\pi G R^2}$$

We get

$$\delta = -\frac{3k}{8\pi G \bar{\rho} R^2}$$

And if we only pay attention to evolving quantities (R and ρ) we can think about how δ evolves:

$$\delta \sim \frac{1}{\bar{\rho} R^2} \sim \frac{1}{R^{-3} R^2} \sim R$$

So the fluctuation grows as R grows. But we also know that scale factor and redshift are related by $R = 1/(1+z)$, so rewrite this in terms of redshift:

$$\delta \sim (1+z)^{-1}$$

Evolution of density fluctuations (continued)

So if $\delta \sim (1+z)^{-1}$, we can relate the density fluctuations at two different times (“initial” and “final”) by the expression:

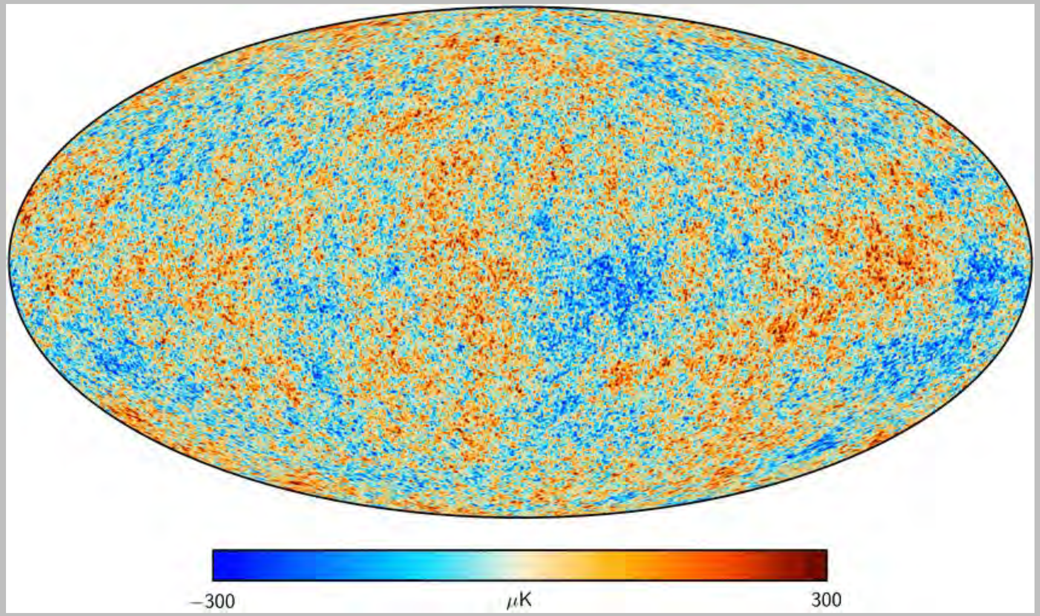
$$\frac{\delta_f}{\delta_i} = \frac{(1+z)_i}{(1+z)_f}$$

If we start with $\delta_i \approx 10^{-5}$ at $z \approx 1000$, by a redshift of $z \approx 5$, the overdensity should have grown to be:

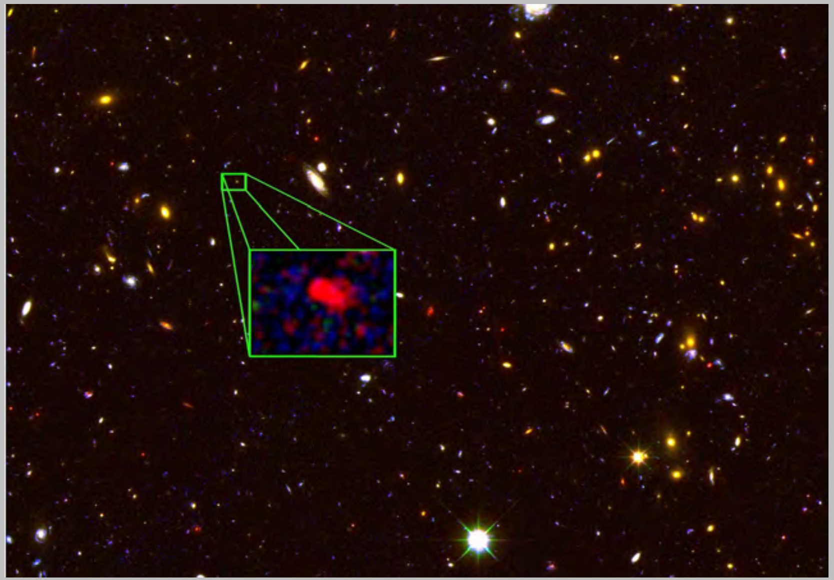
$$\delta_f = \delta_i \frac{(1+z)_i}{(1+z)_f} = 10^{-5}\left(\frac{1001}{6}\right) \approx 0.002$$

Oops! That’s off by many orders of magnitude. By themselves, these fluctuations in baryonic mass ***aren’t strong enough*** to grow into galaxies and galaxy clusters we see at high redshift.

What have we forgotten? ***Dark matter***.



Initial values: $\delta_i \approx 10^{-5}$ at $z_i \approx 1000$



Final values: $\delta_i \approx 10^{5}$ at $z_f \approx 5$

Dark Matter and Structure Formation

Non-baryonic dark matter does not interact with other particles or photons in any way but through gravity.

Dark matter overdensities could grow freely well before recombination, and there's more dark matter than baryonic matter.

So strong gravitational potential wells were already in place for the baryons to collapse into once recombination occurs.

Once an overdense region gets to $\delta > 1$, its dense enough to govern its own growth, and it decouples from the overall expansion of the Universe. It gravitational collapse happens roughly on a free-fall timescale:

$$t_{ff} \approx \sqrt{1/G\rho}$$

Object	Density (ρ) $M_\odot\, pc^{-3}$	Overdensity (δ)
Globular Cluster	100	10^9
Galaxy	10^{-3}	10^4
Galaxy Cluster	10^{-5}	100

Note: Low mass things have higher densities and are the first to form! Massive things form later.

Flavors of Dark Matter

Baryonic Dark Matter

Examples: Faint brown dwarfs, planets, diffuse gas clouds, free-floating space donkeys

Two fatal problems we have already discussed:

- Can’t form structure fast enough because they can’t grow until after recombination.

- Ruled out by big bang nucleosynthesis arguments ($\Omega_b \ll \Omega_m$)

Baryonic dark matter models do not work.

Non-baryonic dark matter

Classified by the characteristic random velocities (energies):

- **Hot dark matter**: particles moving at relativistic speeds
- **Cold dark matter**: particles moving at much slower speeds

Flavors of Dark Matter

Hot Dark Matter: Particles moving at high relativistic speeds
Example: neutrino

In the early universe, HDM particles moving at relativistic speeds will quickly escape from low mass density fluctuations. These fluctuations will no longer be bound, and will not collapse. The only fluctuations that survive are things with masses $\gtrsim 10^{15}\ M_{\odot}$ (massive galaxy cluster scales). These take a long time to collapse, since they are low density.

Once they collapse, and the density increases, smaller structures can start to collapse inside them, a process called "fragmentation". (The way individual stars form inside a collapsing gas cloud)

So in Hot Dark Matter models:

- **Structure forms "top down"**: big things first, then smaller and smaller things.
- **Structure forms slowly**: Have to wait for the big low density things to collapse before structure can form.
- **Galaxies form late** in the Universe's history.

This is not what we see – hot dark matter models do not work!

Flavors of Dark Matter

Cold Dark Matter: Particles moving at low speeds
Example: NO KNOWN OBJECTS

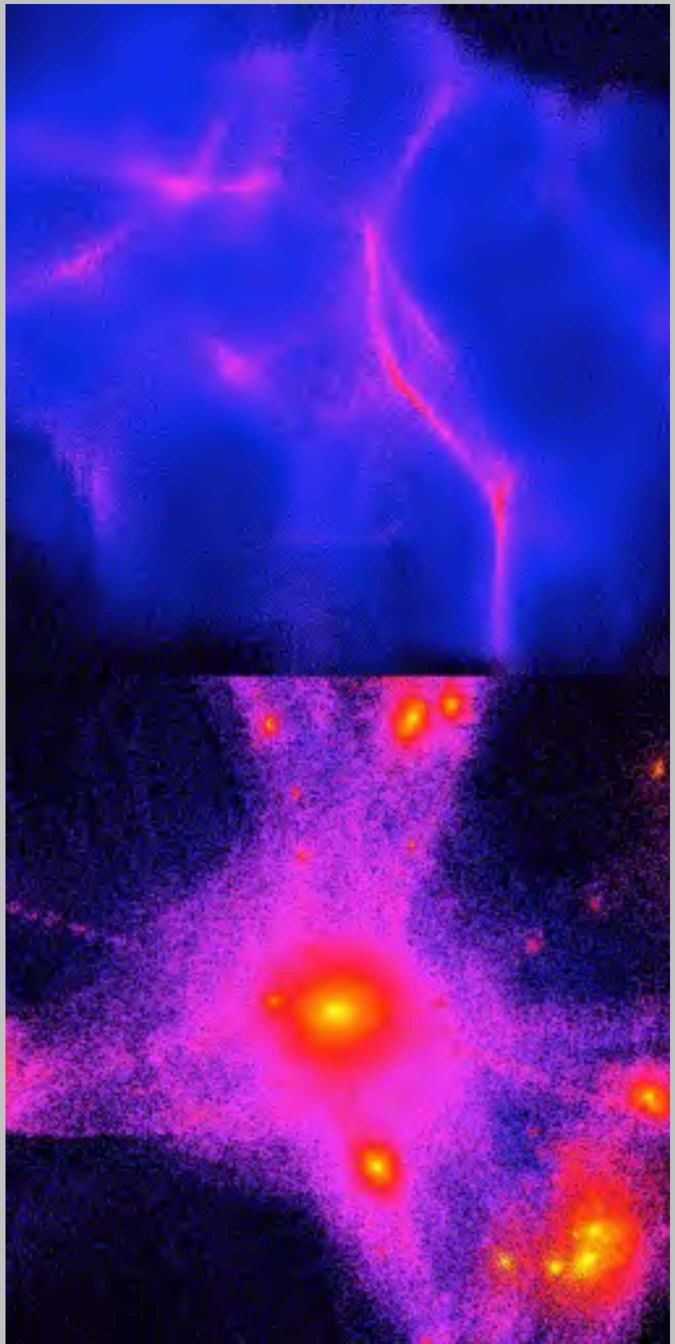
In the early universe, CDM particles move much more slowly and can be bound into low mass "halos" of dark matter. Once recombination occurs, baryons collapse into these low mass halos first, then over time low mass halos (of dark matter and baryons) continue to merge on larger and larger size scales: Hierarchical formation.

So in Cold Dark Matter models:

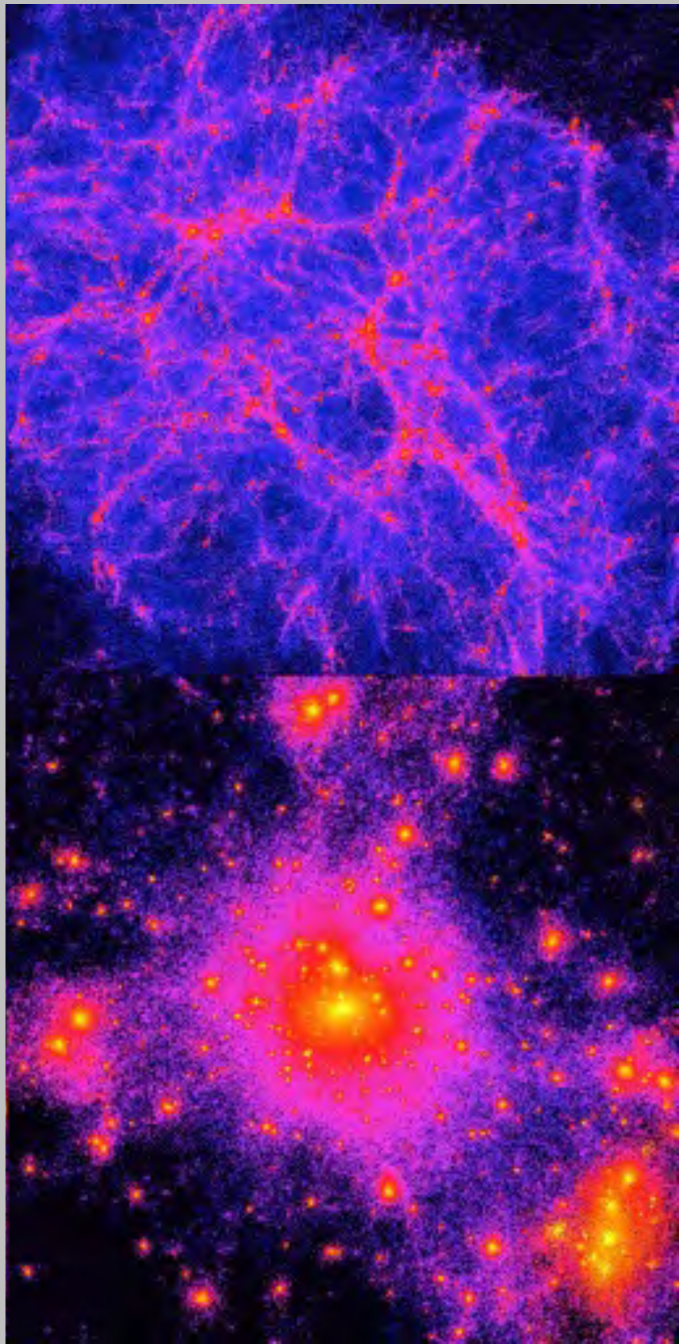
- **Structure forms "bottom up"**: small things form first, then merge together over time to form bigger things.
- **Structure forms early**: As soon as recombination hits, structure can begin forming quickly
- **Galaxies form early, galaxy clusters form later**.

This is a much better model for what we see: galaxies forming in the early Universe, galaxy clusters growing at later times.

Hot Dark Matter



Cold Dark Matter



HDM: very little structure early;

CDM: much more early structure.

Early Universe ⟺

HDM: no dwarf galaxies at late times.

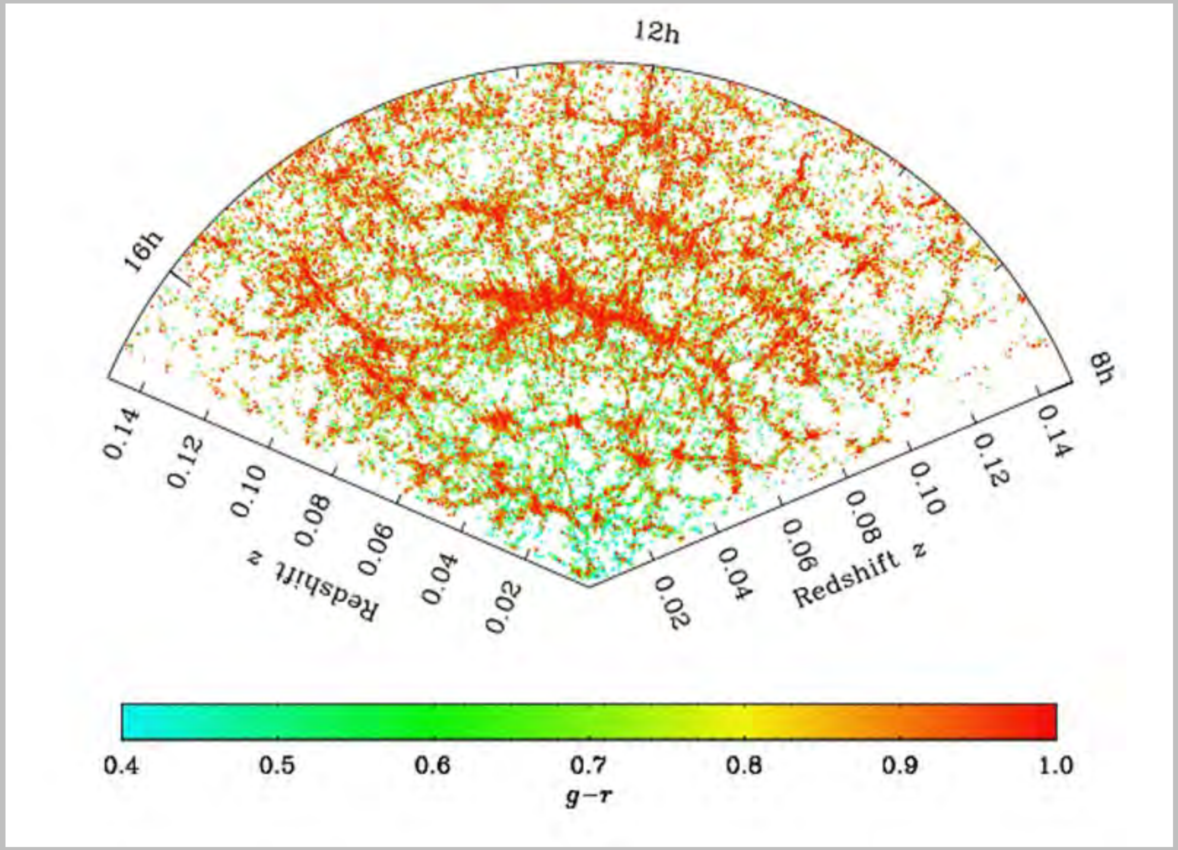
CDM: many more low mass galaxies at late times.

Today ⟺

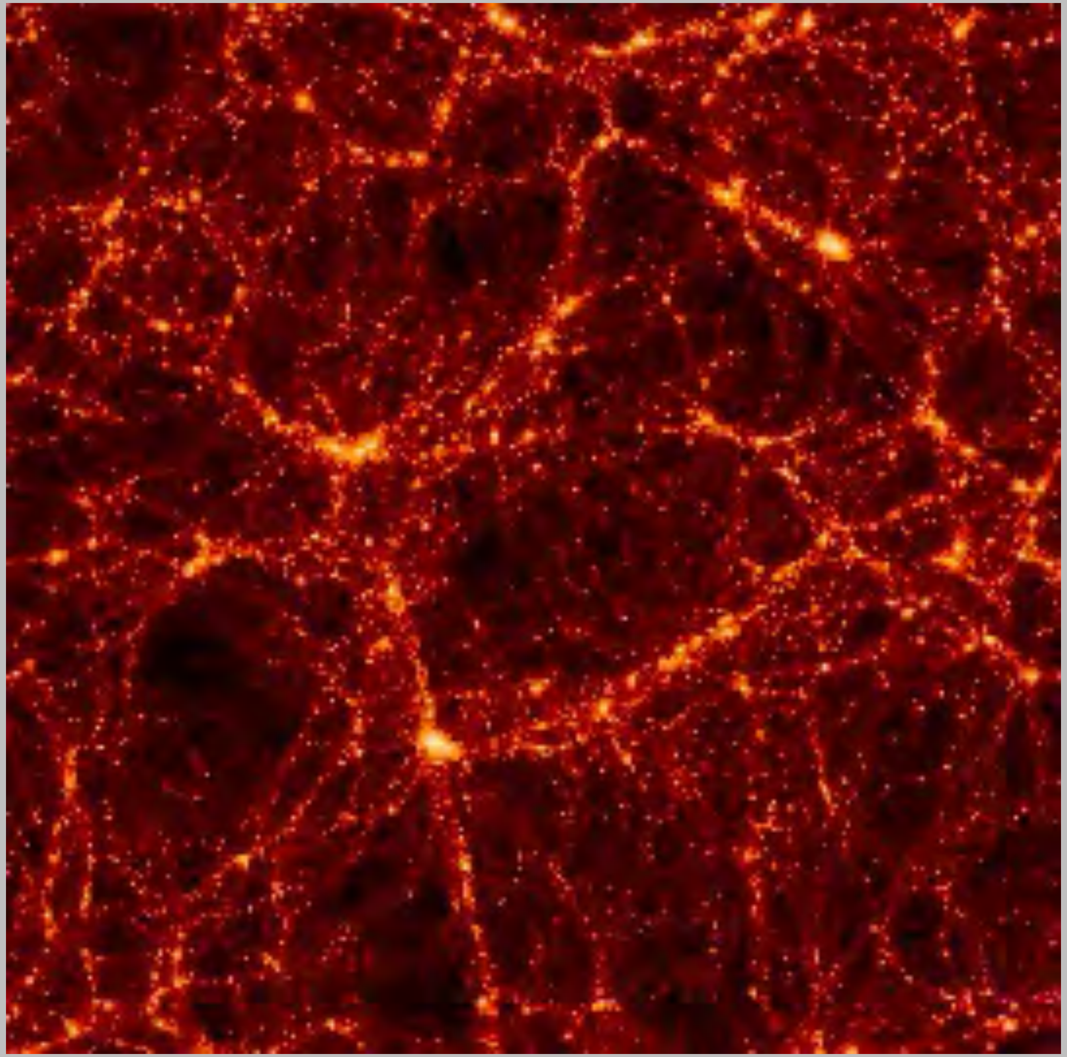
Simulations of structure formation
(courtesy ITC/Zurich)

Structure Formation under Cold Dark Matter models

Connecting observations of large scale structure.....



...to predictions from theoretical models



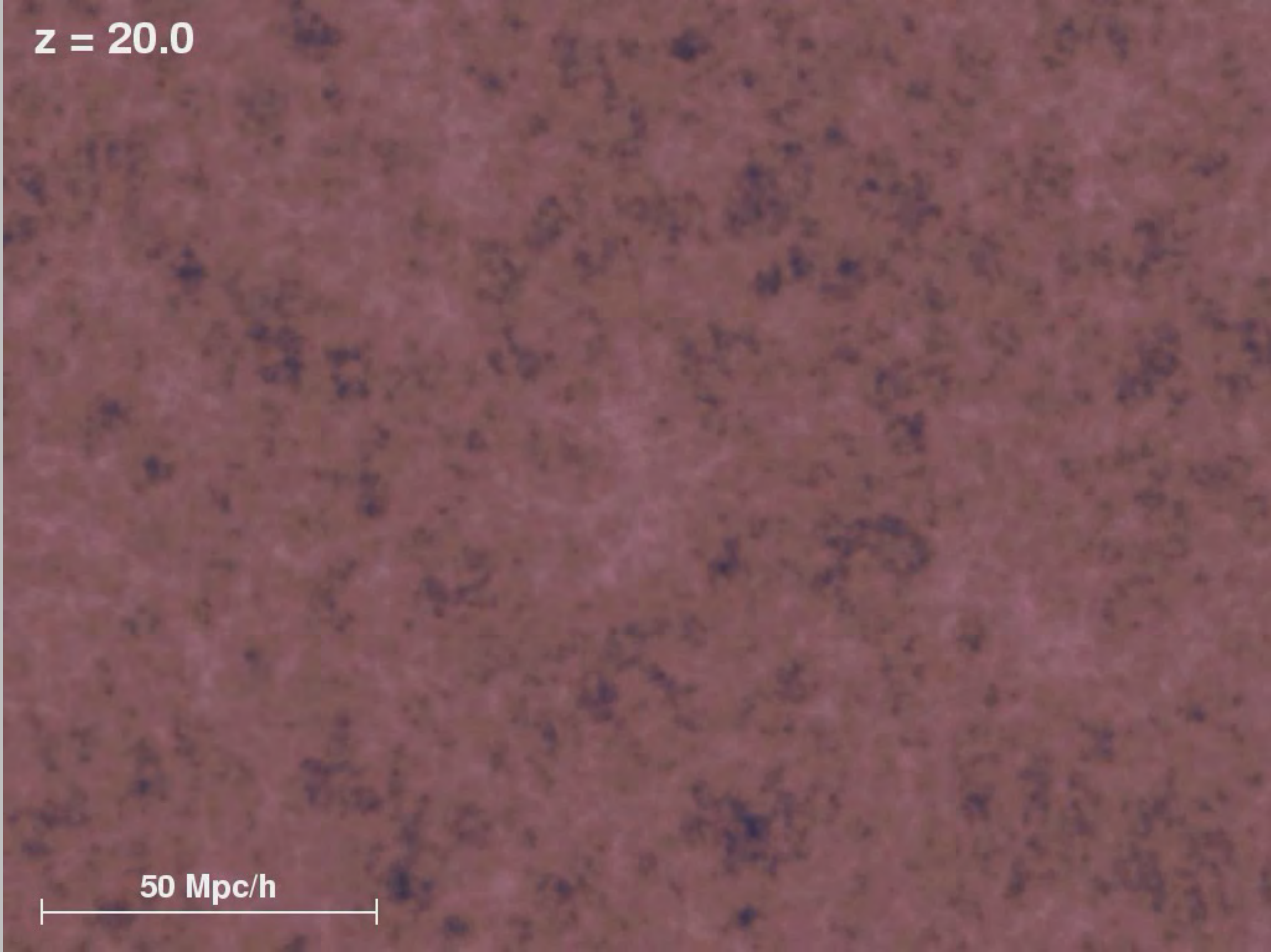
The formation of structure

Small fluctuations in the mass density at early time grow in strength due to gravity pulling mass together.

Mass forms filamentary structure, collects in dense regions at the intersections of filaments.

Large voids also grow as matter empties out of them into the filaments.

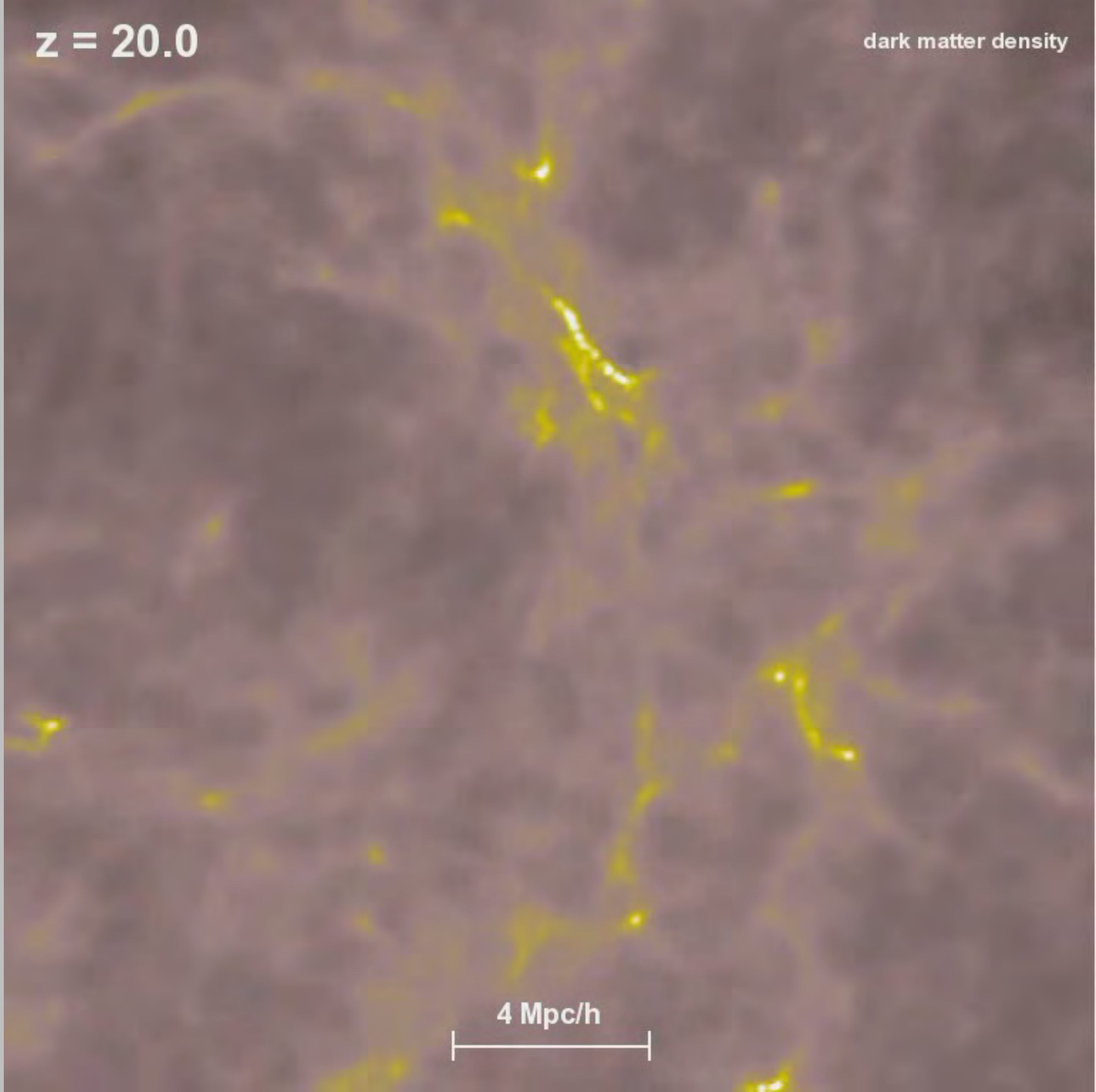
Dark matter simulation from the Millenium Simulation (Springel+05)



Forming Galaxy Clusters

In cold dark matter models, clusters form hierarchically: small lumps merge together to form bigger lumps, which merge to form even bigger lumps, etc.

Clusters grow over time, and the rate at which they grow depends on the density of the universe and its expansion history.



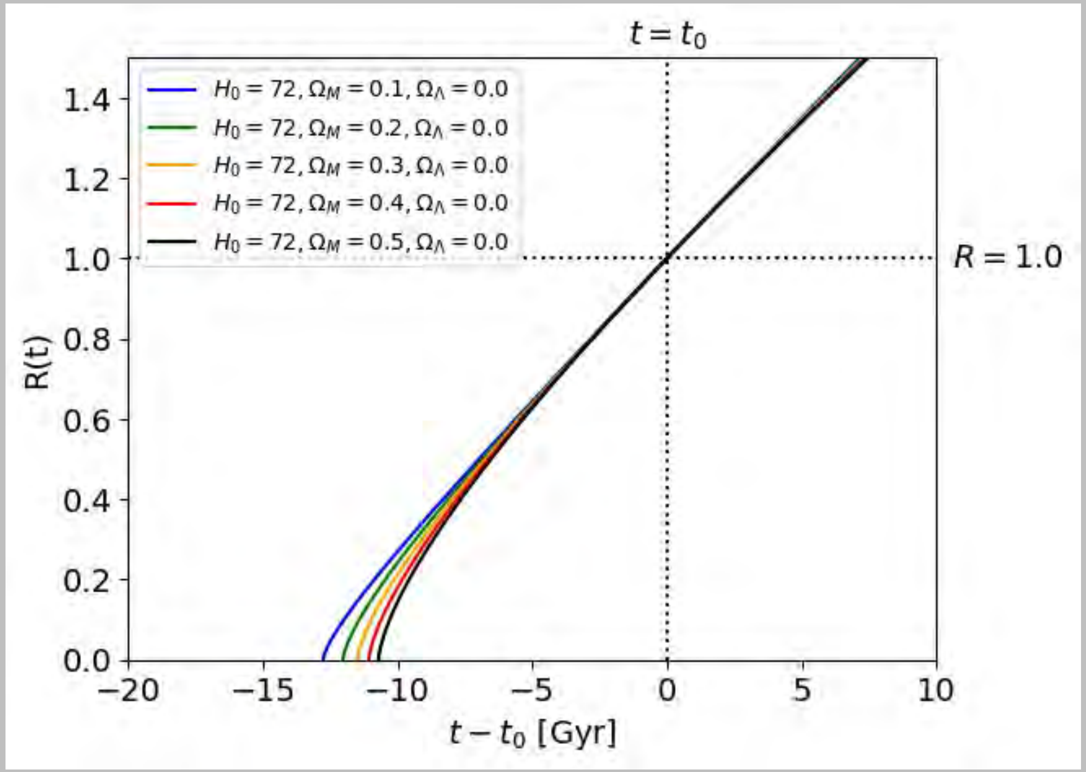
Dark matter simulation from the Millenium Simulation (Springel+05)

Structure Formation under Cold Dark Matter models

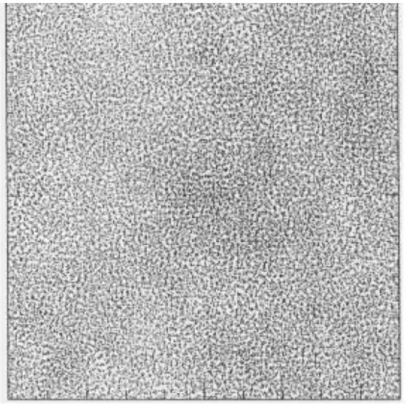
First, just consider Universes where there is no dark energy, so $\Omega_\Lambda = 0.0$.

The more mass there is (i.e., bigger Ω_m) the more structure there is at present day. More mass ⇒ stronger gravity ⇒ structure grows faster.

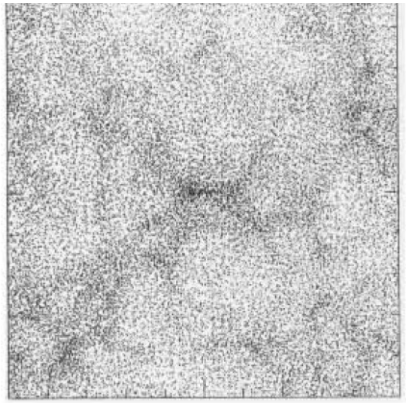
Structure seems "right" at values of $\Omega_m \approx 0.4$ or so.
But to get the age right, we want $\Omega_m \lesssim 0.2$. *That's a problem!*



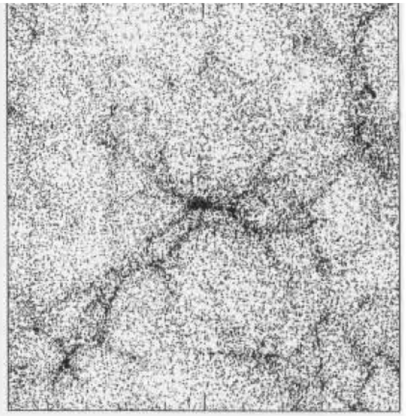
$\Omega_m = 0.1$



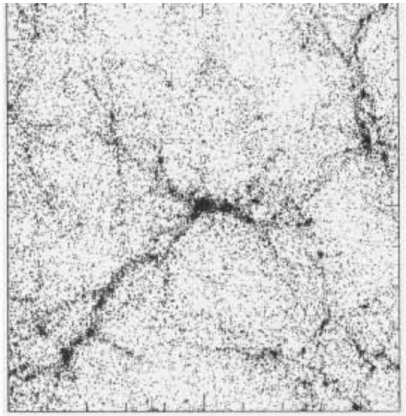
$\Omega_m = 0.2$



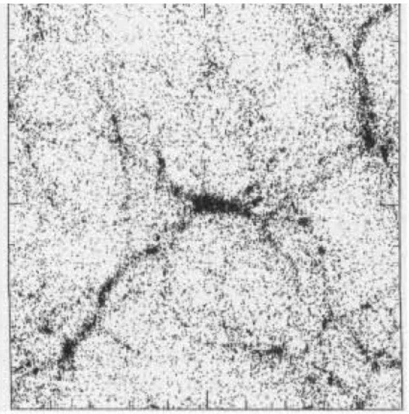
$\Omega_m = 0.3$



$\Omega_m = 0.4$



$\Omega_m = 0.5$



Simulated present-day universes.

All models have $\Omega_\Lambda = 0.0$

Cole+97

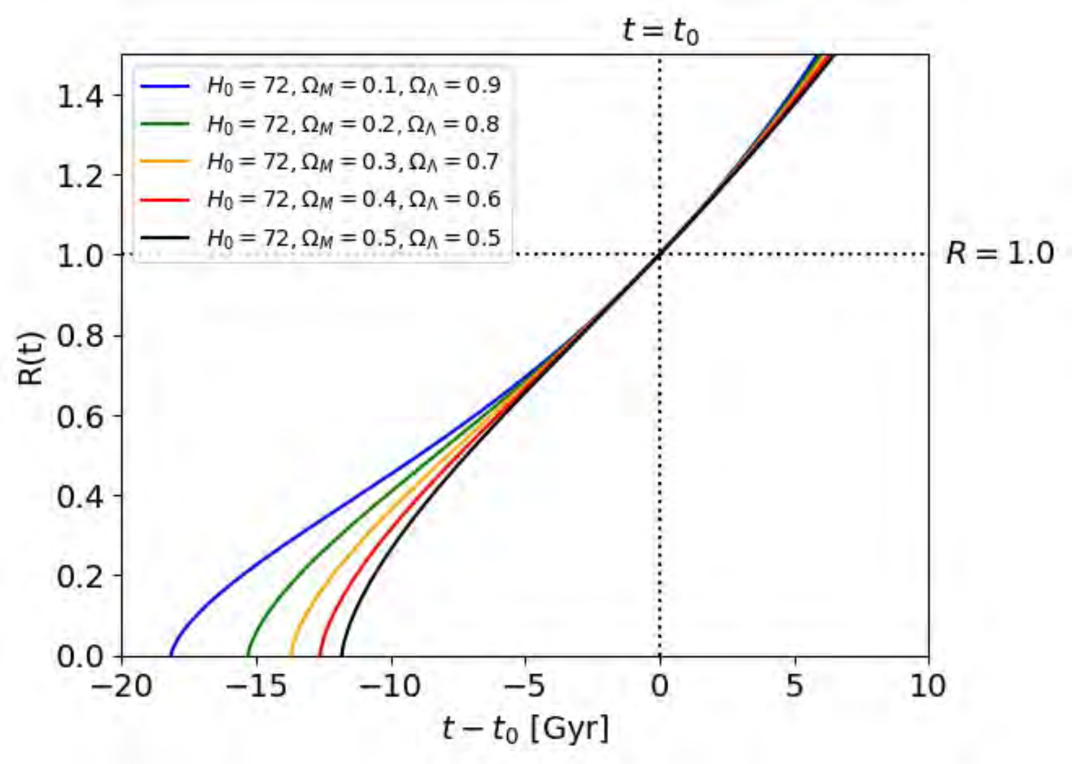
Structure Formation under Cold Dark Matter models

Now, just consider flat Universes where $\Omega_m + \Omega_\Lambda = 1.0$.

At fixed Ω_m, universes have much more structure than before (when we held $\Omega_\Lambda = 0$).

Dark energy makes universes older, so more time for structure to grow.

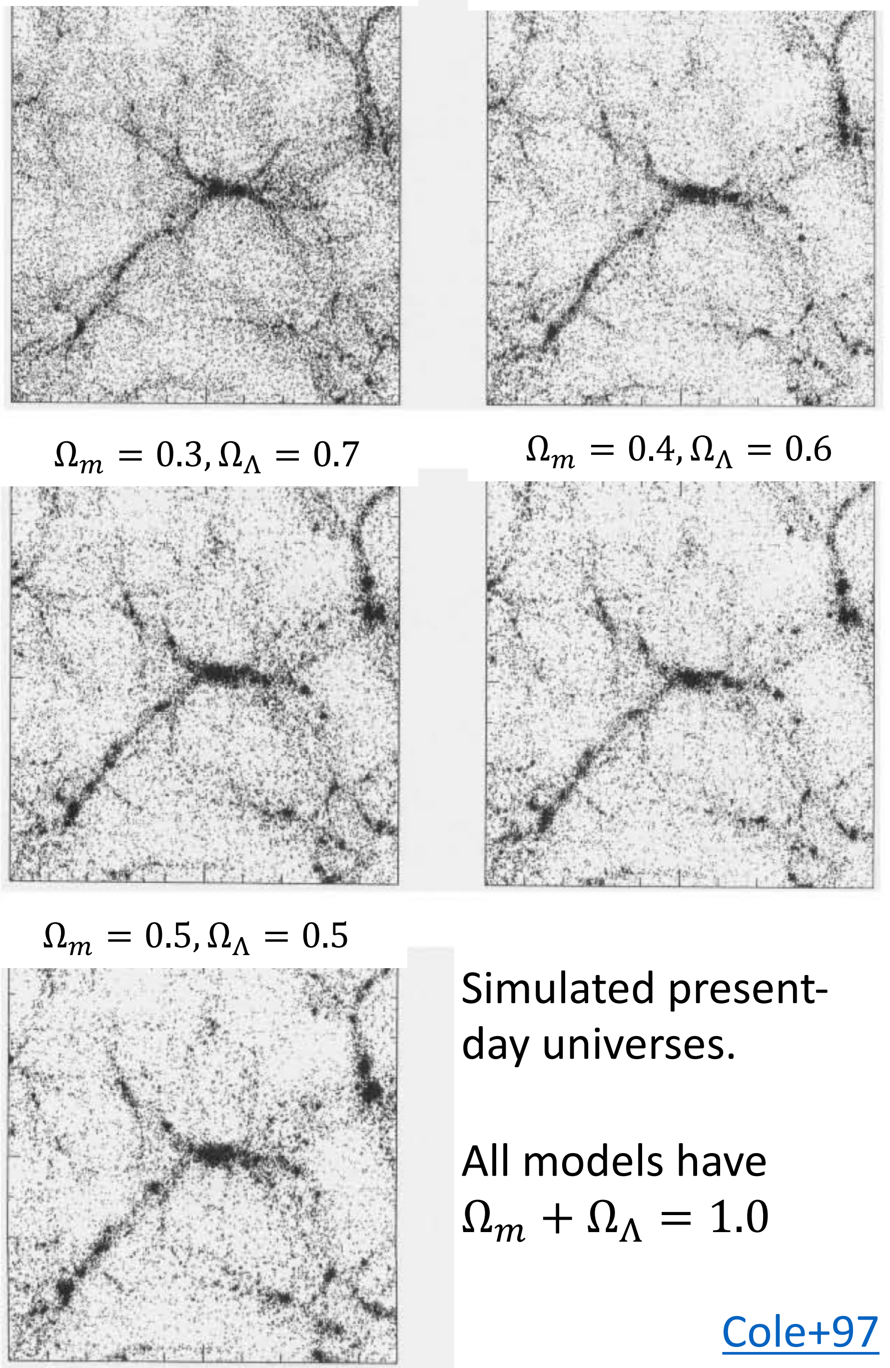
Get a good match to structure *and* age at $\Omega_m = 0.3, \Omega_\Lambda = 0.7$



$\Omega_m = 0.1$ $\Omega_\Lambda = 0.9$ $\Omega_m = 0.2, \Omega_\Lambda = 0.8$

$\Omega_m = 0.3, \Omega_\Lambda = 0.7$ $\Omega_m = 0.4, \Omega_\Lambda = 0.6$

$\Omega_m = 0.5, \Omega_\Lambda = 0.5$



Simulated present-day universes.

All models have $\Omega_m + \Omega_\Lambda = 1.0$

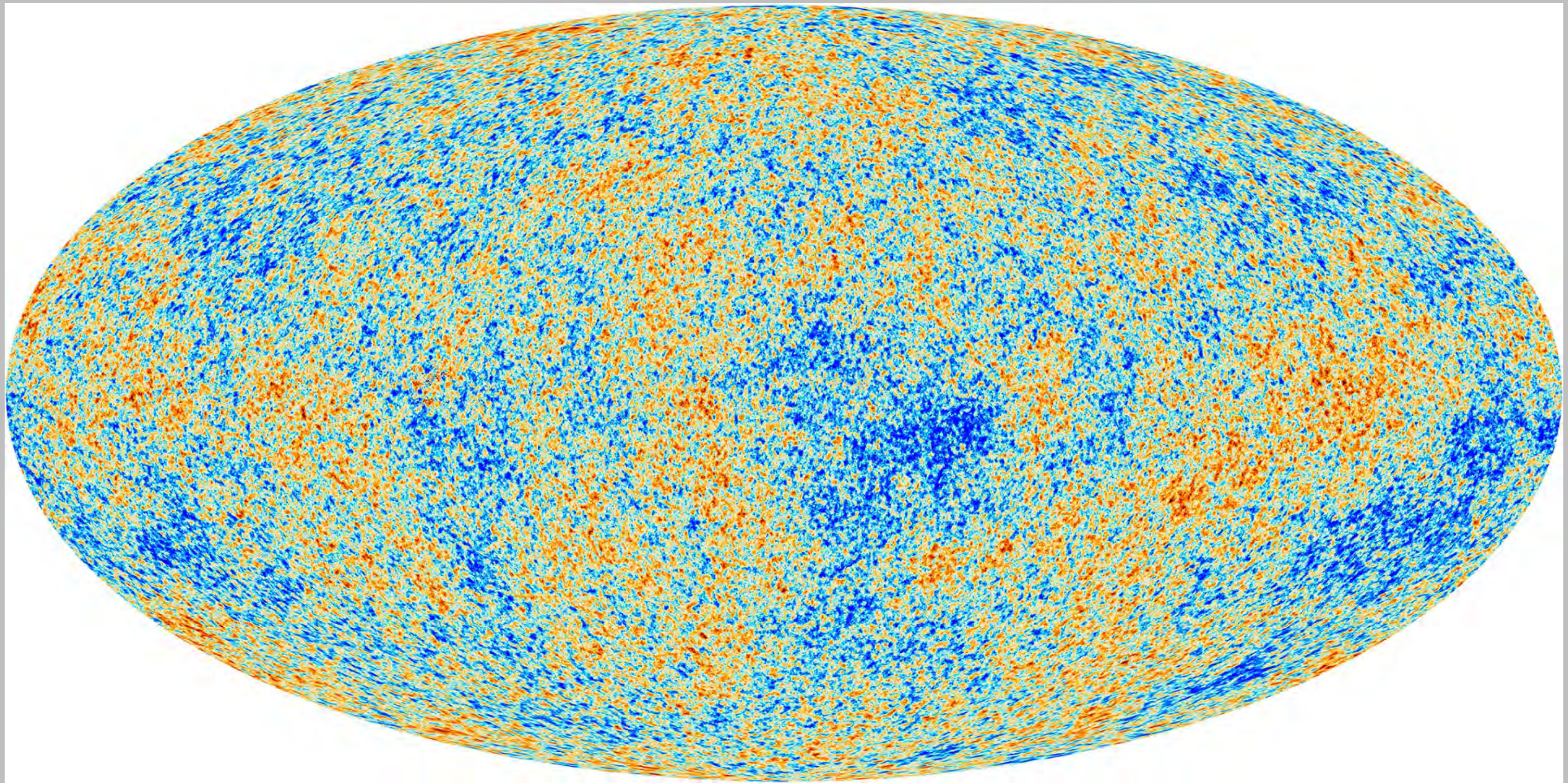
Cole+97

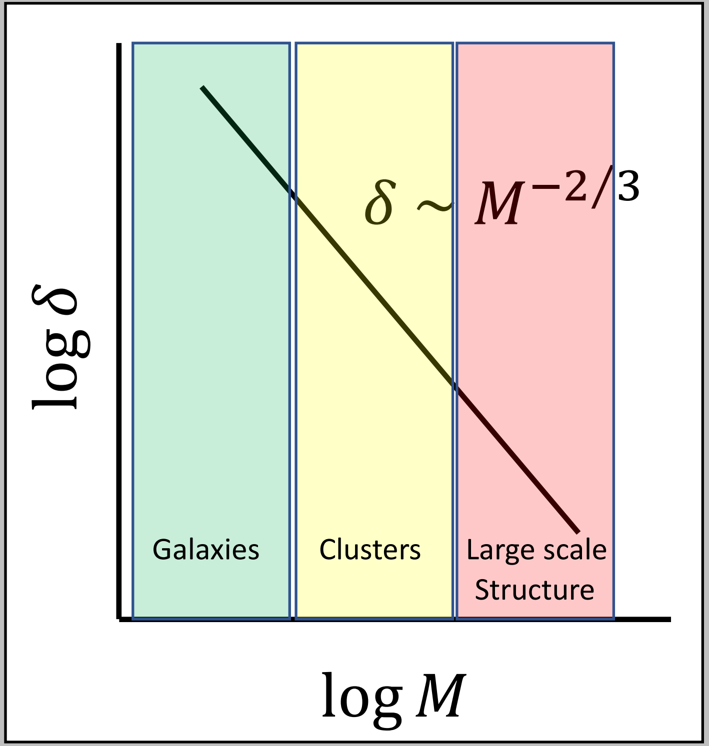
Growing Galaxies

Remember that the fluctuations in mass density traced by the microwave background have characteristics sizes of about 65 Mpc -- much larger than galaxies and galaxy clusters. On smaller scales inside those fluctuations (and unresolved by current CMB data) are the fluctuations destined to grow into galaxies.

Fluctuations on smaller mass scales are stronger overdensities ($\delta \equiv \Delta\rho/\rho$). $\Rightarrow$

Low mass things form first, then merge to form large things: ***hierarchical assembly***.





Fluctuation Power Spectrum: More "power" (stronger overdensities) on smaller mass scales.

Hierarchical Growth of Galaxies and Merger Trees

Merger Tree:

- Time runs down the chart
- Width of branches/trunk = galaxy masses
- Branches meeting = galaxies merging.

Early times: many seperate low mass (and gas-rich) galaxies at high redshift.

As time goes by: ongoing mergers, star formation

Late times: Large galaxy in local universe

Questions:

- *When did this galaxy form?*
- *When did this galaxy's stars form?*
- *What do we mean by the age of a galaxy?*

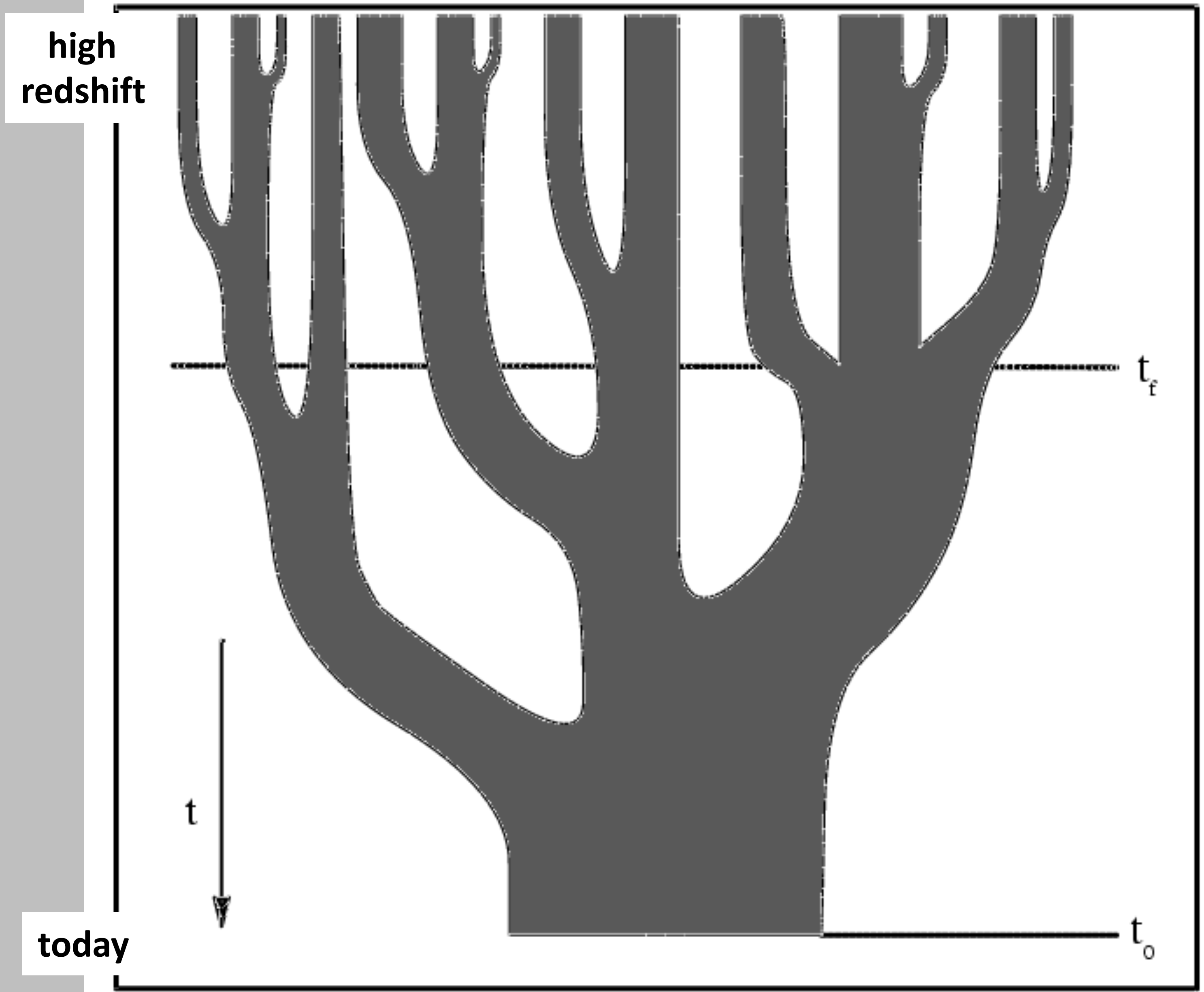


Figure 6. A schematic representation of a "merger tree" depicting the growth of a halo as the result of a series of mergers. Time increases from top to bottom in this figure and the widths of the branches of the tree represent the masses of the individual parent halos. Slicing through the tree horizontally gives the distribution of masses in the parent halos at a given time. The present time t_0 and the formation time t_f are marked by horizontal lines, where the formation time is defined as the time at which a parent halo containing in excess of half of the mass of the final halo was first created.

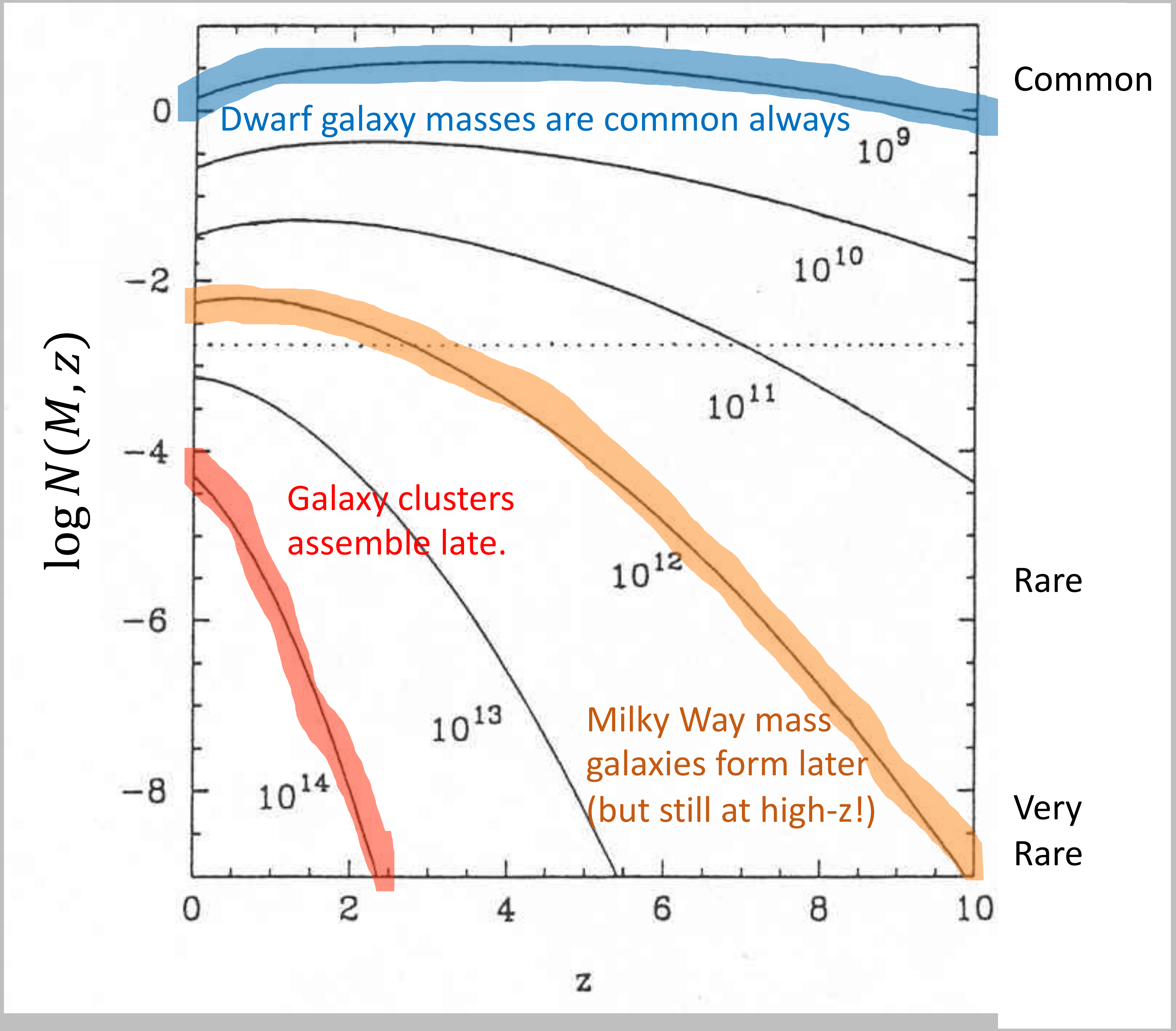
Dark Matter Halo mass function

Plot the (log) number (N) of dark matter halos of a given mass (M) as a function of redshift (z).

As low mass galaxies merge together to form higher mass halos, massive halos become more common.

Predictions:

- massive galaxies should be rare in the very early universe ($z > 5$).
- galaxy clusters should be rare at $z > 1$ (first half of Universe's history).



Simulating Galaxy Evolution

Is very hard! The ingredients:

- Cosmological initial conditions
- Gravitational dynamics
- Gas hydrodynamics
- Radiation / Photoionization
- Star formation, supernovae, stellar winds
- AGN triggering, radiation, and outflows
- Stars and Stellar Evolution
- Dust absorption / extinction

Requires “High Performance Computing”

Example: The Illustris Project, international collaboration of astronomers, physicists, and computer scientists..

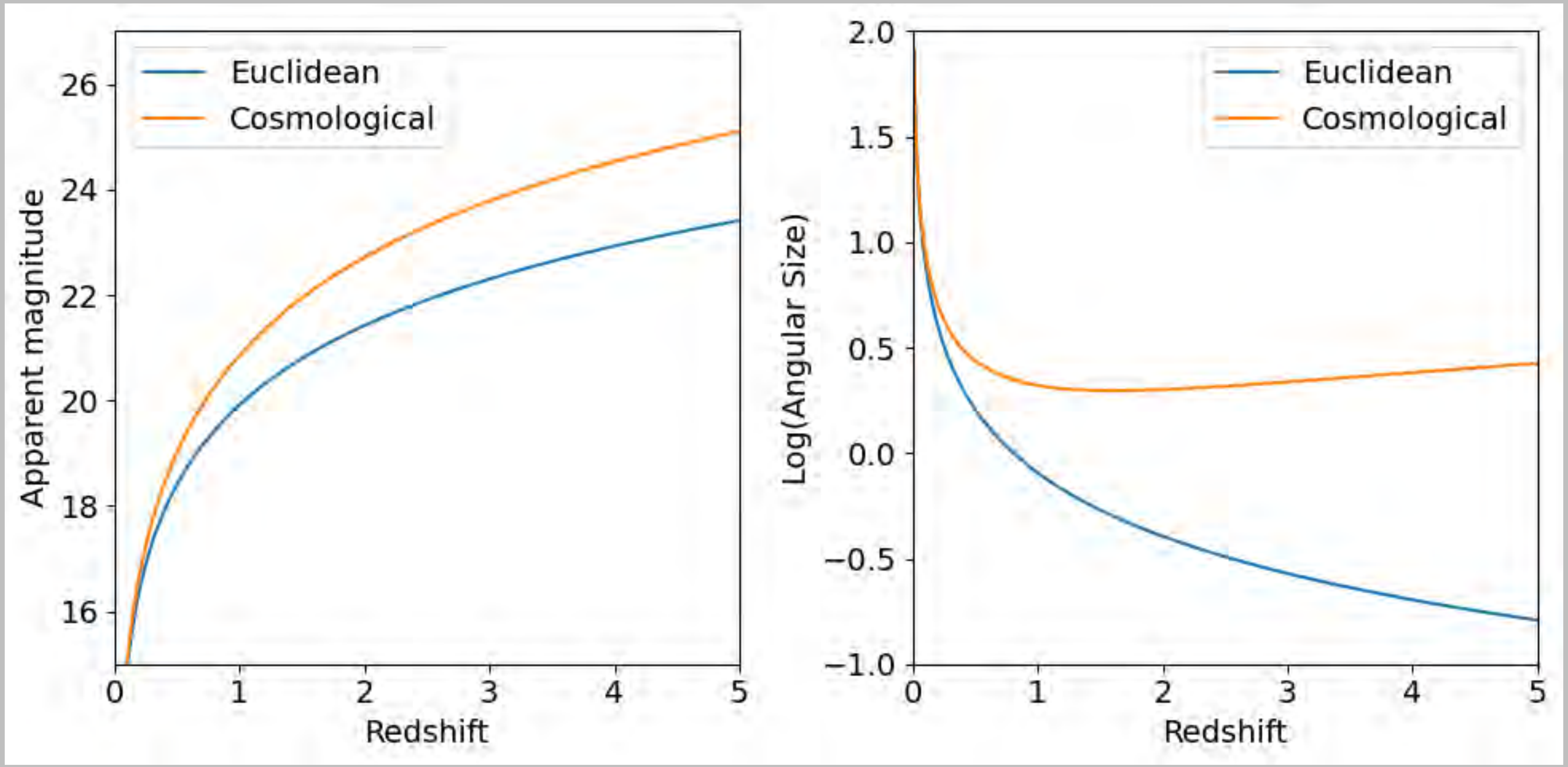
Largest simulations run on 8,192 compute cores, and took 19 million CPU hours (the equivalent of one computer CPU running for 19 million hours, or about 2,000 years

Simulations continue to get bigger and include even more physics, resolveing smaller and smaller scales, and doing so over larger and larger volumes. Some of the most intensive computational tasks in modern science.

Observing Galaxies in the Early Universe

Compared to the local universe "Euclidean expectation", if we think of moving a galaxy to higher and higher redshifts, cosmological effects make it

a) even fainter in apparent magnitude
b) not appreciably smaller beyond z=1.



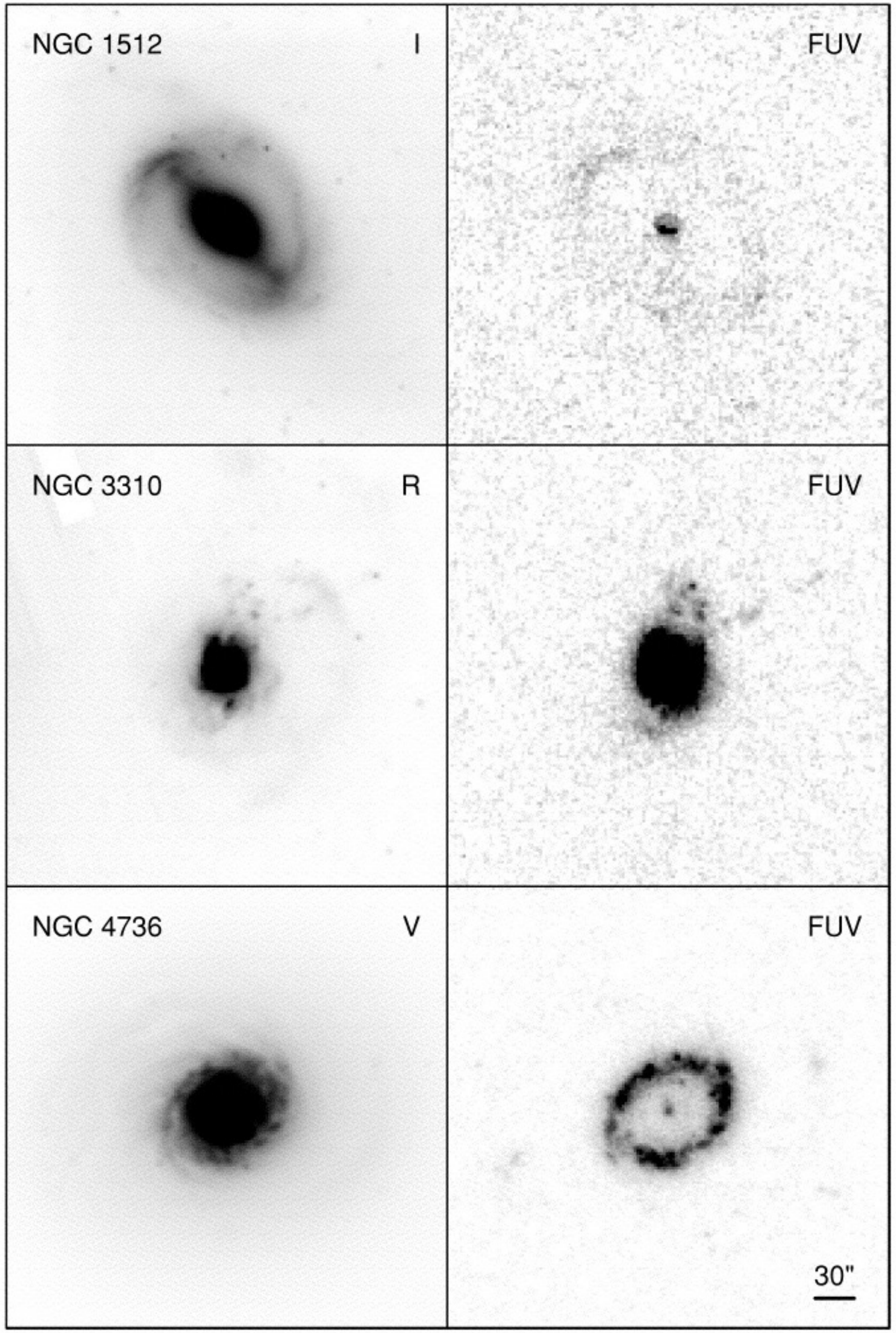
Observing Galaxies in the Early Universe

Also, at high redshift when we observe with optical telescopes, we see redshifted light that was originally emitted by the galaxy in the ultraviolet.

Ultraviolet light:

- dominated by young massive stars, not the general stellar population.
- easily obscured by dust.

Galaxies look very different in the ultraviolet than they do in the visible.



Images of nearby galaxies

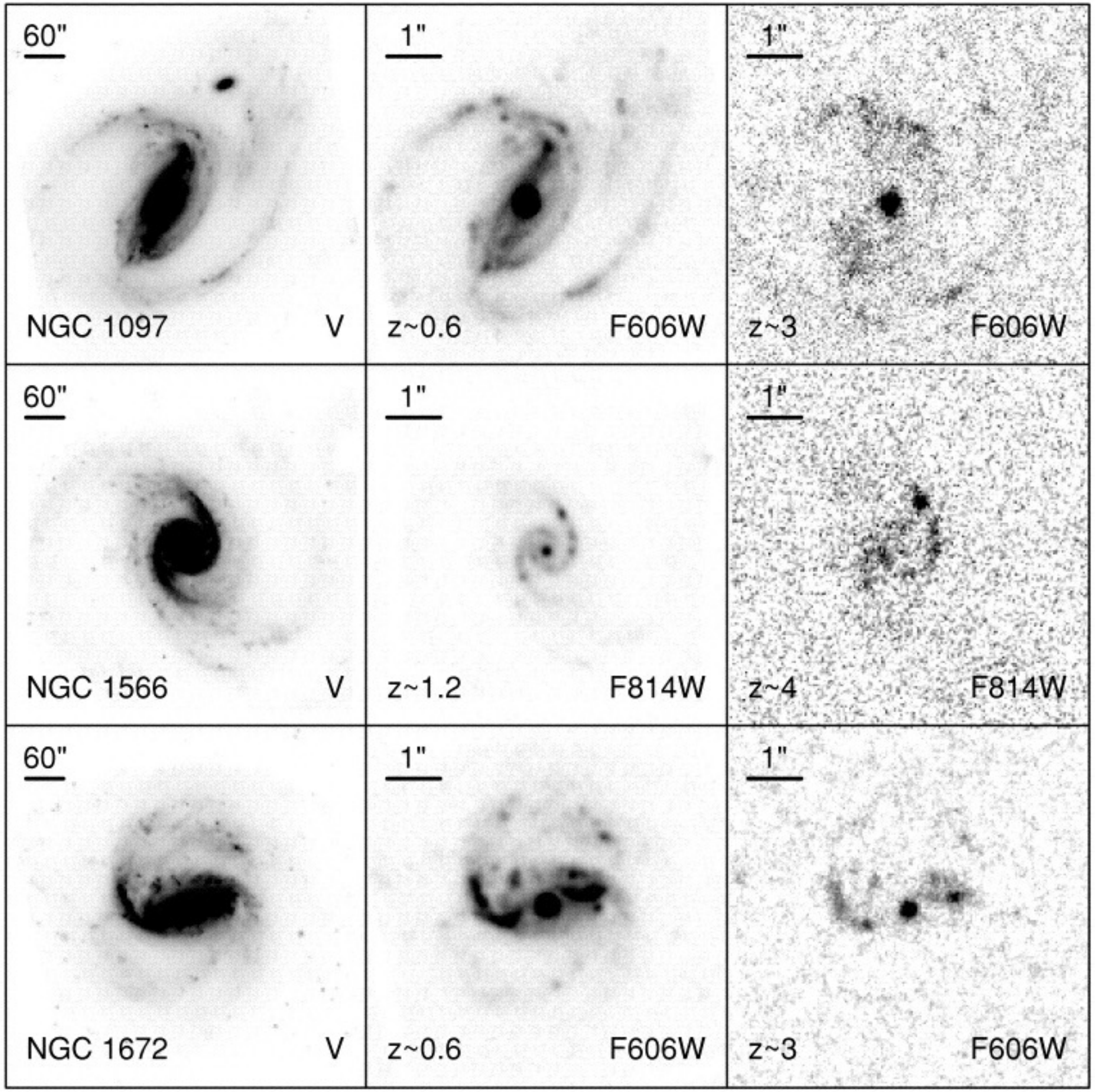
Left column: Optical images
Right column: Ultraviolet images

Kuchinski+01

Observing Galaxies in the Early Universe

So the combination of brightness and size effects, coupled with bandshifting, means that optical images of high-z galaxies can look very different from local galaxies even if they are physically similar!

Tend to only see the highest surface brightness regions and/or the star forming regions of a high redshift galaxy.



“Mock redshifting”

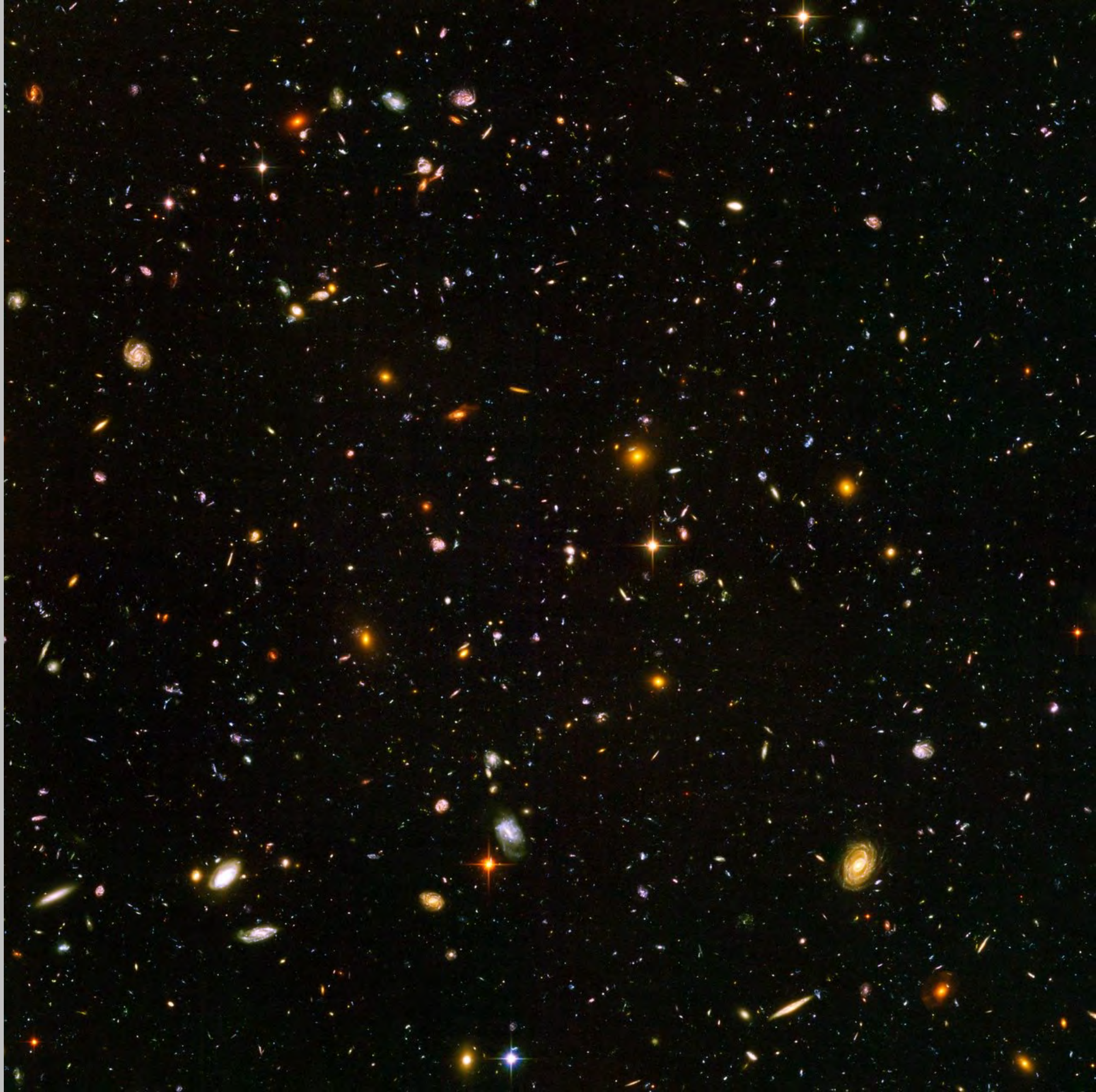
Left: True images of nearby galaxies
Middle: Simulated images at moderately high redshift
Right: Simulated images at high redshift

Kuchinski+01

The Hubble Ultradeep Field

- Observed in 2003-04
- Blank patch of sky 2.5 arcmin across (about $1/10^{th}$ the size of the full moon)
- ~ 1,000,000 seconds of exposure time across four different optical filters
- Deep enough to detect galaxies to z ~ 6-7.

Remember, the image shows all galaxies along the line of sight, across a range of redshifts.



Massive Galaxies in the UDF

Moderate z
$0.5 < z < 1.2$
$\Longleftarrow$

High z
$2.2 < z < 3.0$
$\Longrightarrow$

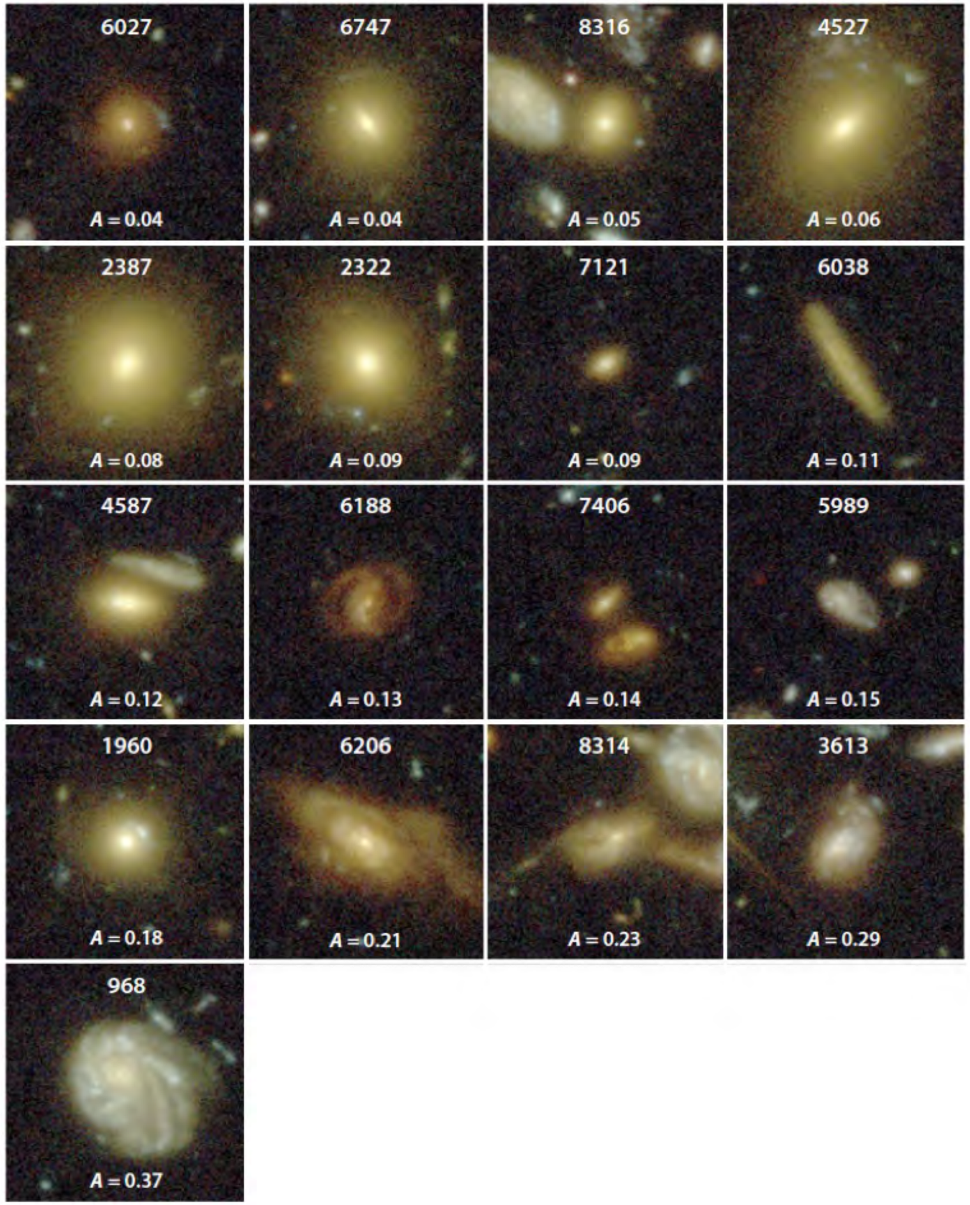


Figure 9

Galaxies in the Hubble Ultra Deep Field as imaged through the ACS camera and ordered by how asymmetric they are. These are all galaxies with redshifts $0.5 < z < 1.2$ and stellar masses $M_* > 10^{10}$ $M_\odot$. The ID is the number used by Conselice et al. (2008), and the A value is the value of the asymmetry. At these redshifts most of the massive galaxies can still be classified as being on the Hubble sequence.

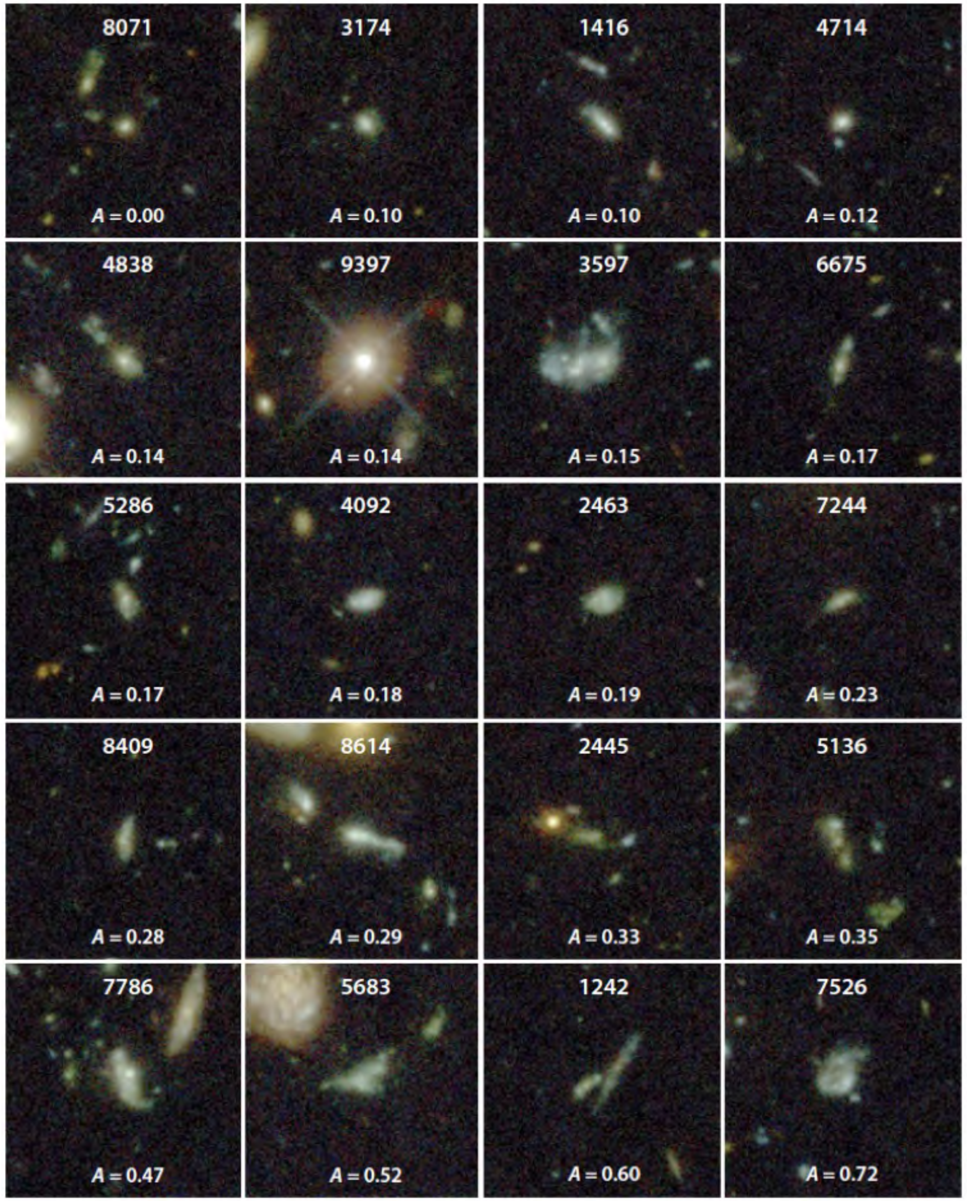


Figure 10

Massive galaxies in the Hubble Ultra Deep Field as imaged through the ACS camera and ordered by the value of their asymmetries from most symmetric to most asymmetric. Shown in this figure are systems with stellar masses $M_* > 10^{10}$ $M_\odot$ at redshifts $2.2 < z < 3$. These galaxies are typically much smaller and bluer and have a higher asymmetry and inferred merger fraction than galaxies of comparable mass today (Conselice et al. 2008).

Conselice ARAA 2014

Galaxy populations evolve with time

Late-type galaxies: spirals
Early-type galaxies: E/S0

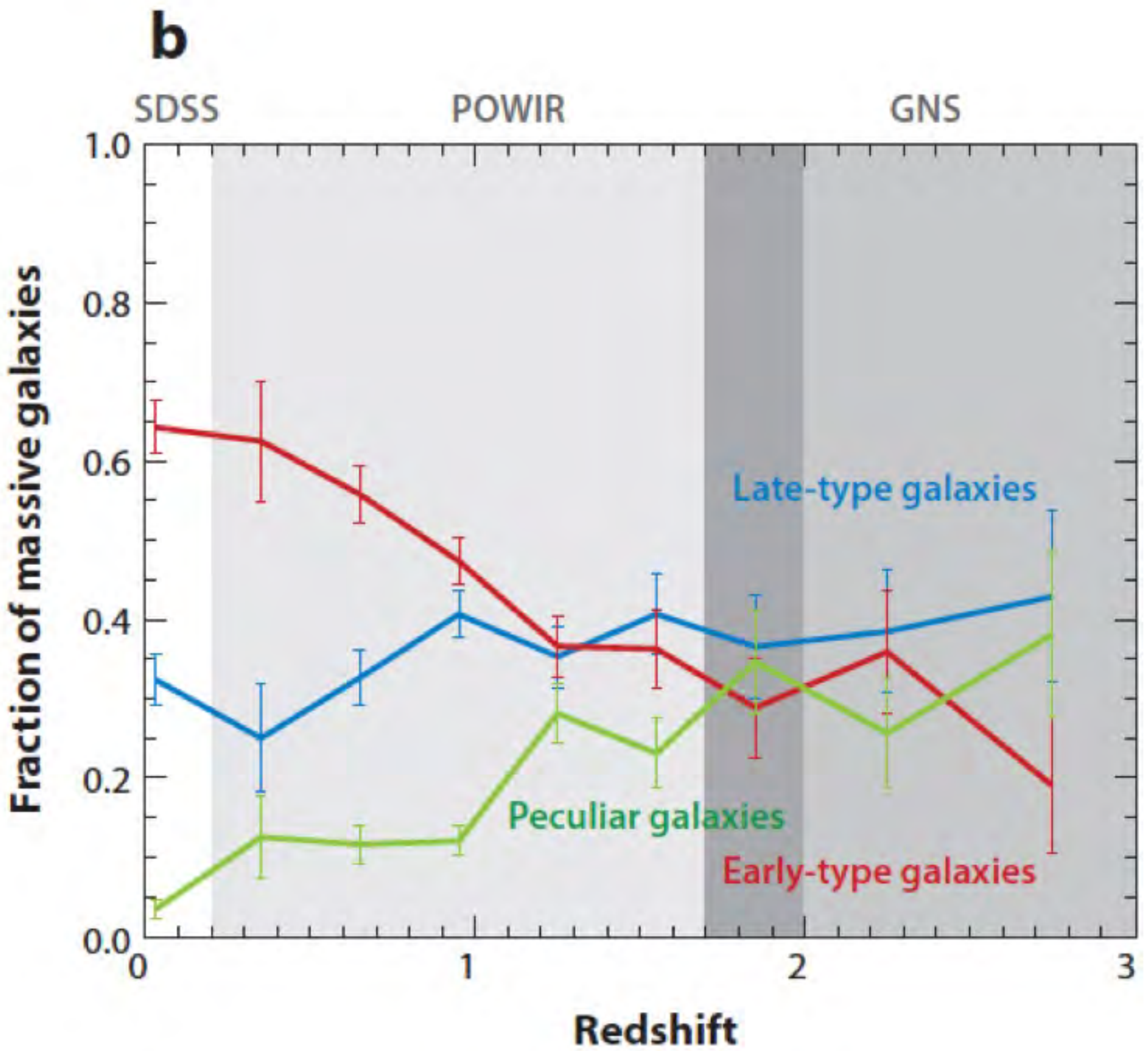
Lots of peculiar galaxies at early times (high redshift), products of interactions, mergers, and starbursts. Density of the Universe was higher, interactions common.

Spirals common across time.

Ellipticals become the dominant type of massive galaxy at late times.

Hierarchical growth shaping the local galaxy population observed today.

[Conselice ARAA 2014]



“Downsizing”

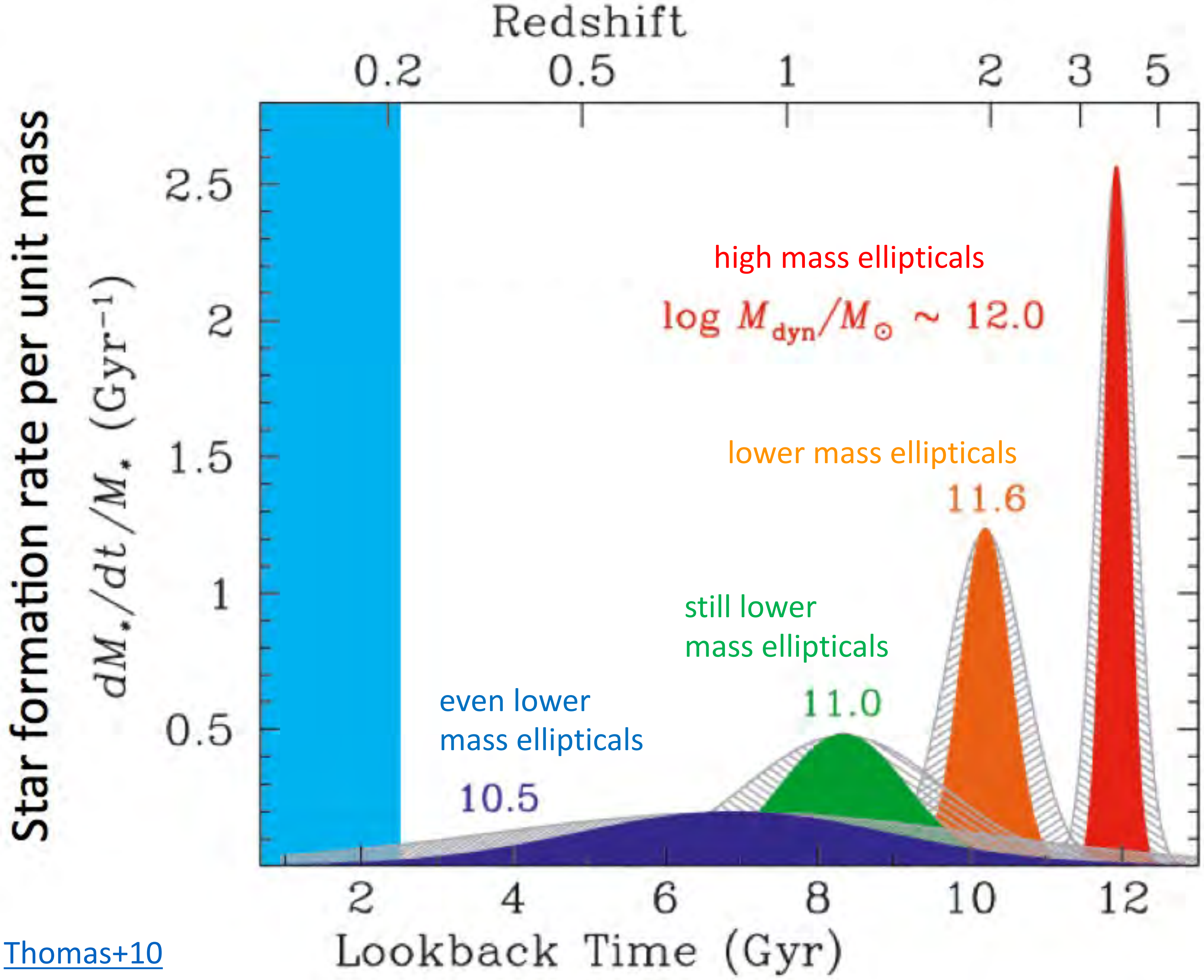
The stellar populations of high mass ellipticals formed very early on.

The stellar populations of lower mass ellipticals formed at later times (but still long ago!)

Remember: the age of the stellar populations is not the same as the age of the galaxy.

The stars in today massive ellipticals formed long ago, in smaller galaxies, that merged together over time to build up the massive ellipticals..

Environmental density important: massive ellipticals are found in dense environments. Their stars formed early, mergers built the ellipticals quickly.



Thomas+10

Inside-out Galaxy Formation

Galaxies are growing in physical size over time.

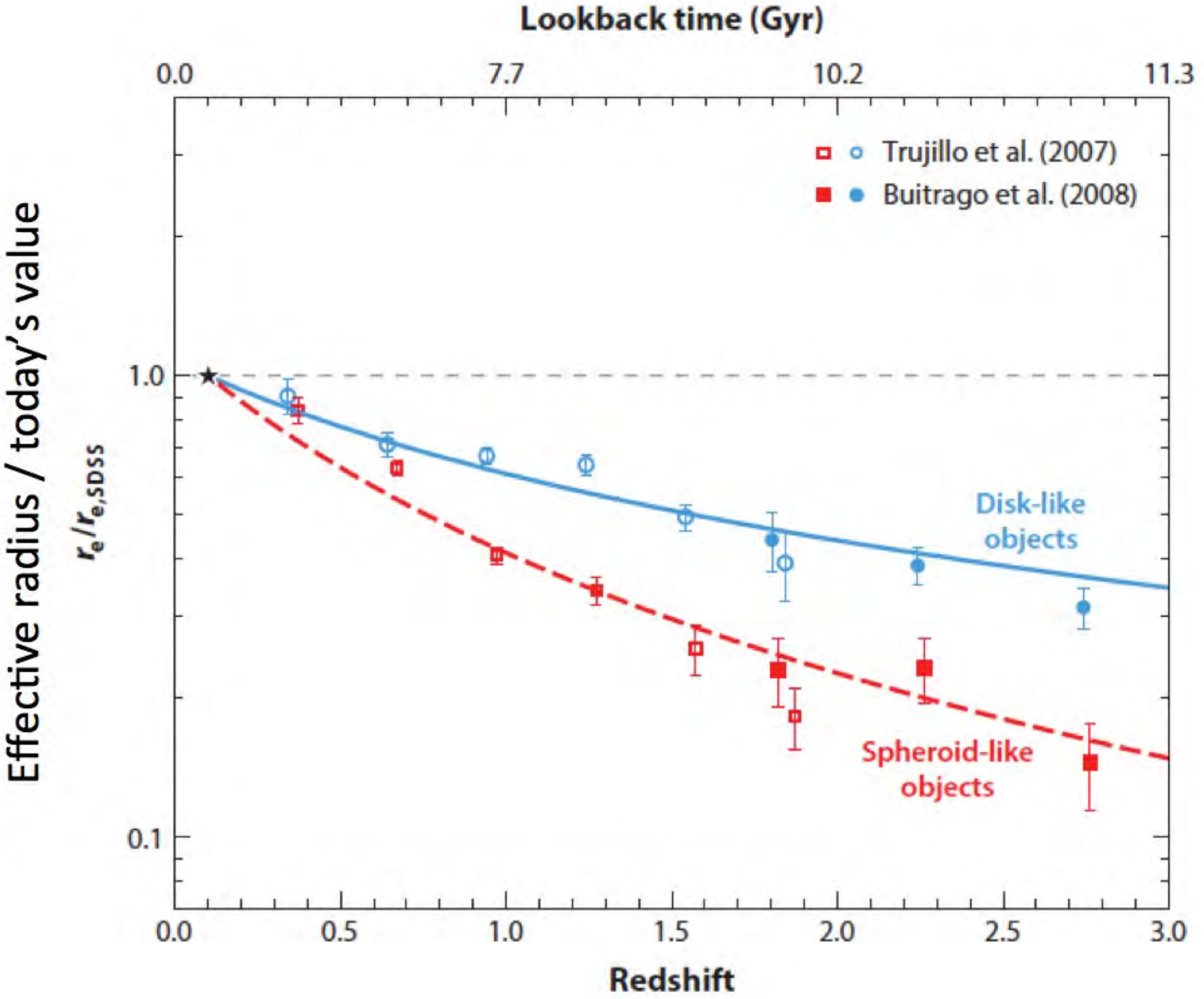
At high redshift galaxies are more compact than today.

Two likely effects:

- Star formation in outer parts of spirals happens more gradually, builds up the outer disks.
- Low mass galaxies fall in to larger galaxies ("accretion") and their stars are stripped and left in the larger galaxies' halos.

Both processes build the outer parts of galaxies.

[Conselice ARAA 2014]



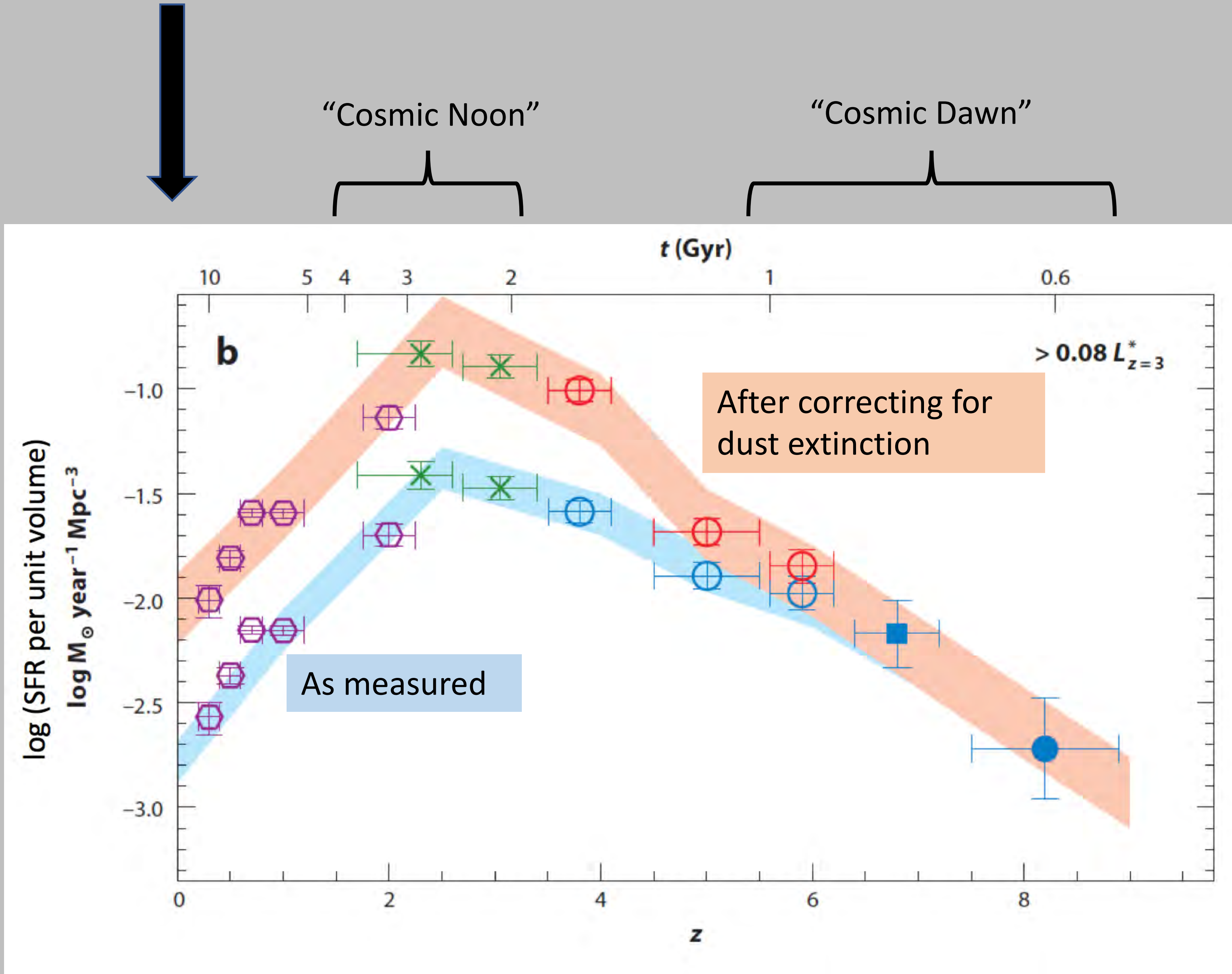
Star forming history of the Universe

Cosmic Happy Hour! 🍸🍺🍷

Integrated over all galaxies, we see that the star formation rate of the universe grows quickly and peaks at $z \approx 2 - 4$, or a few billion years after the Big Bang.

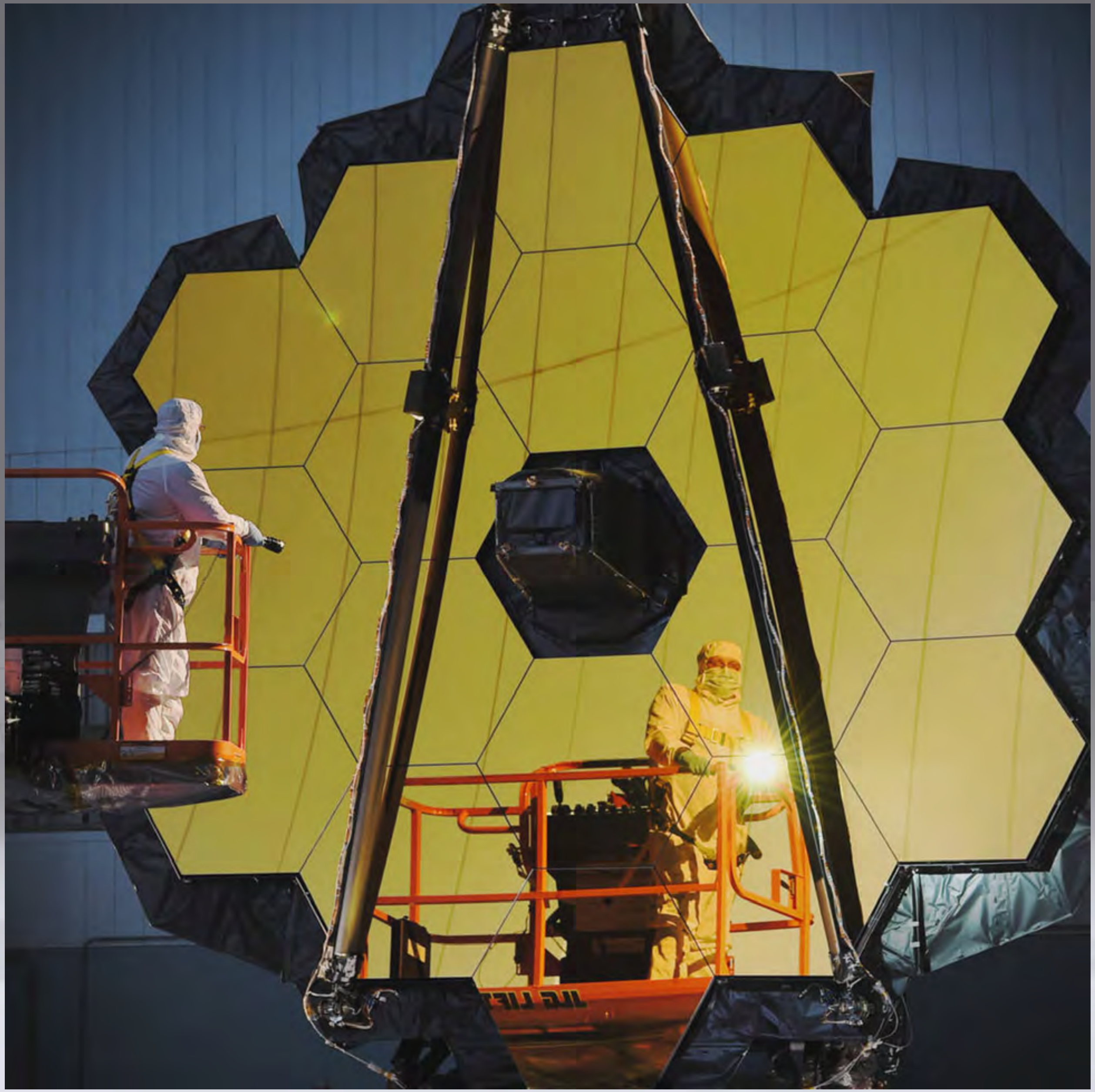
Since them star formation has been slowly ramping down.

Individual galaxies may behave differently, of course!

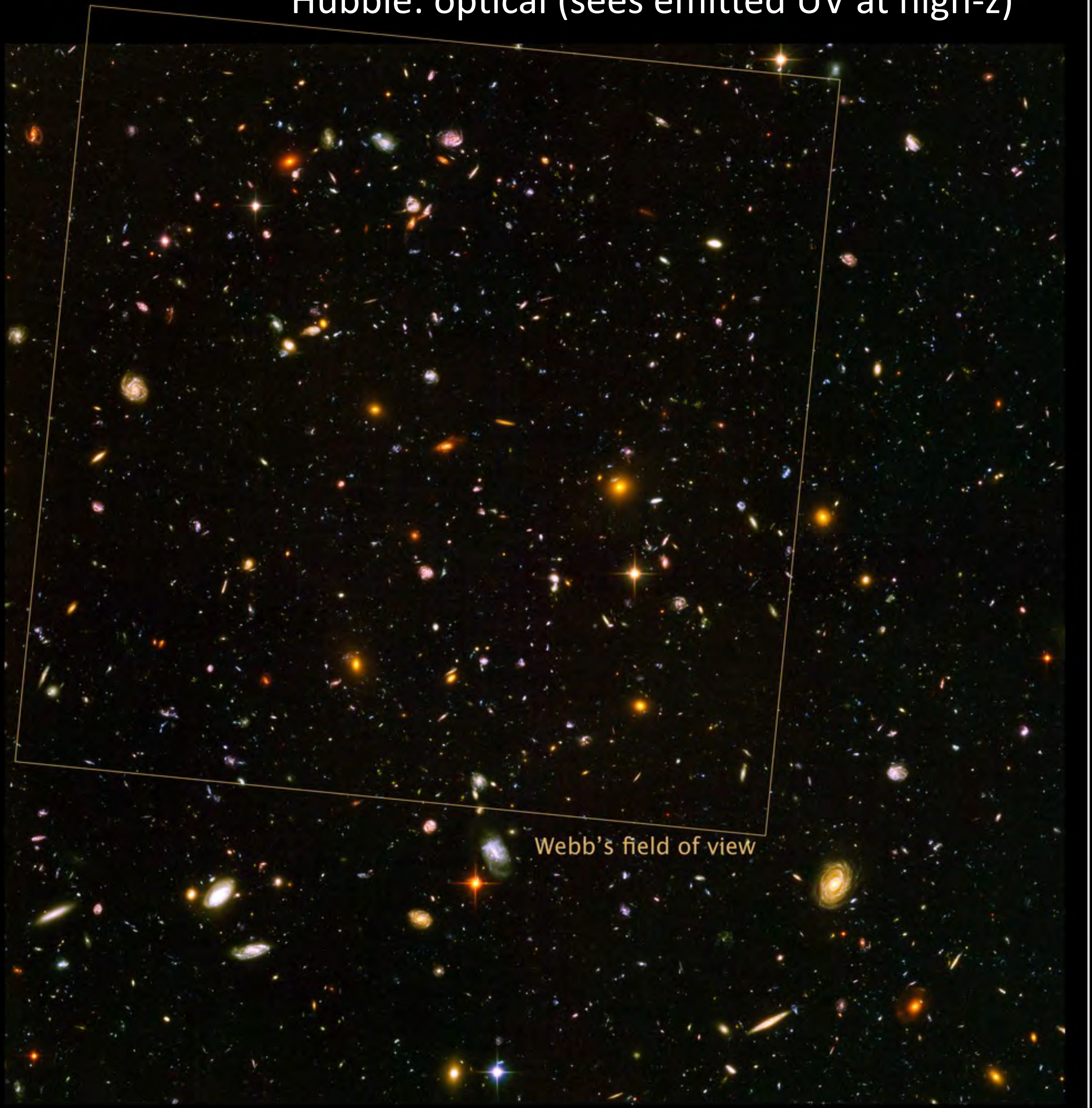


JWST vs Hubble

JWST



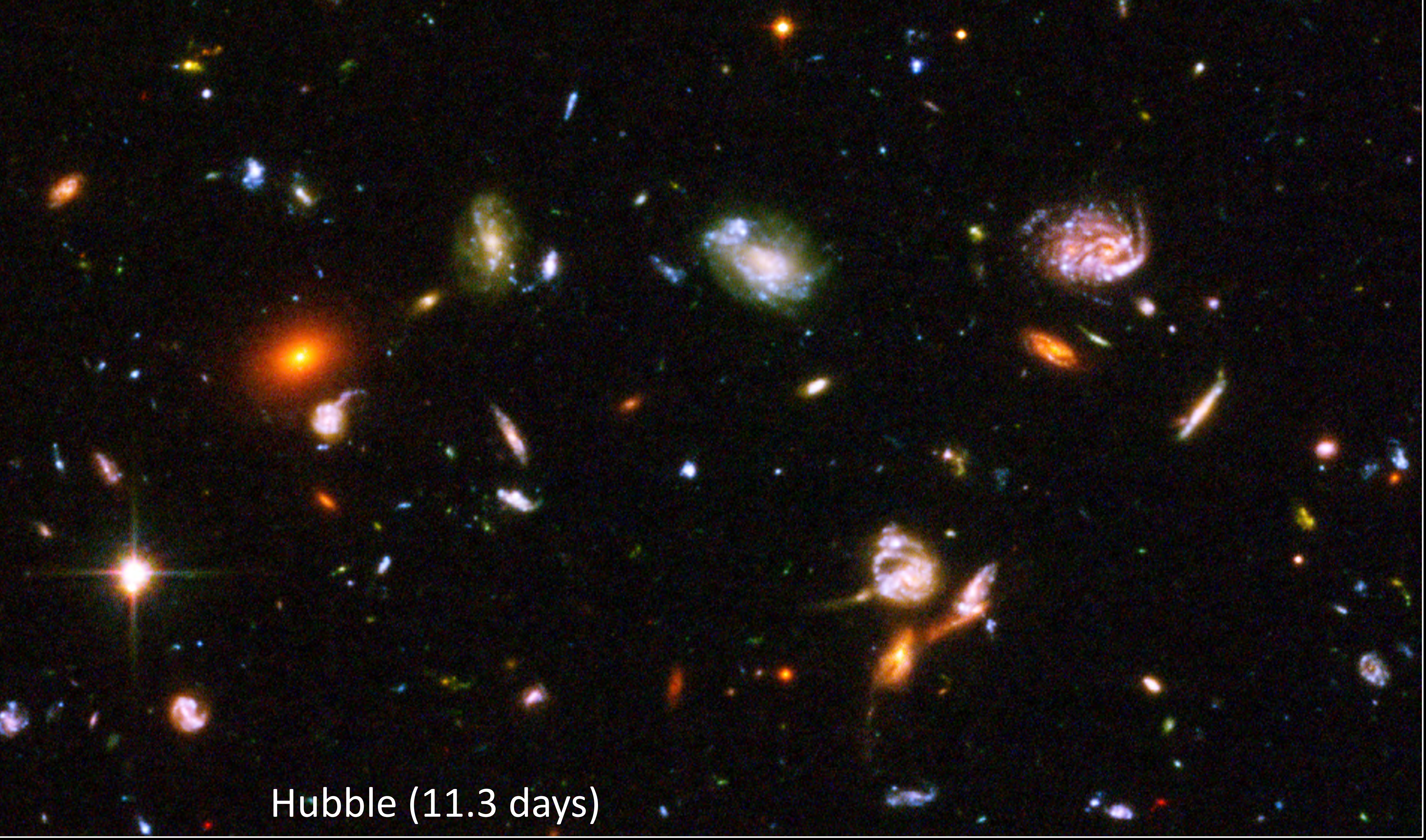
Hubble: optical (sees emitted UV at high-z)



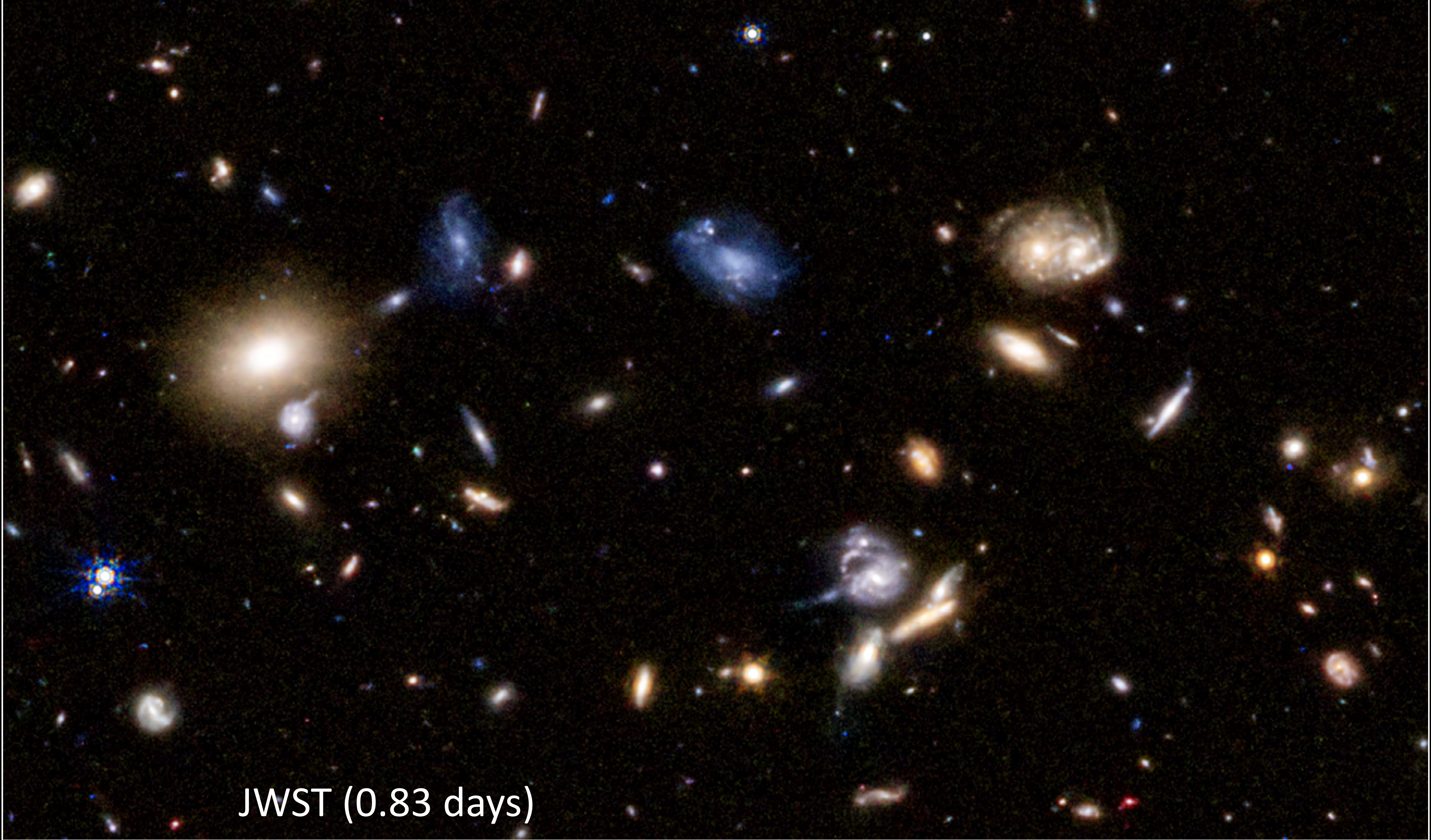
Hubble UDF (exposure time: 11.3 days)

JWST: infrared (sees emitted optical at high-z)

Webb (exposure time: 0.83 days)



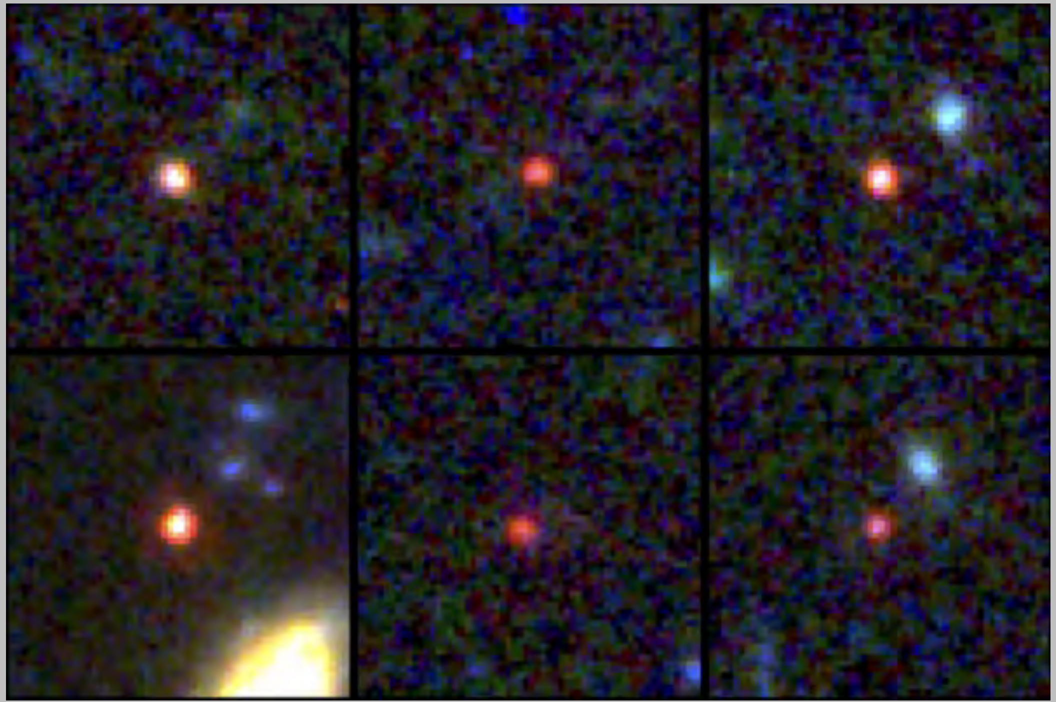
Hubble (11.3 days)

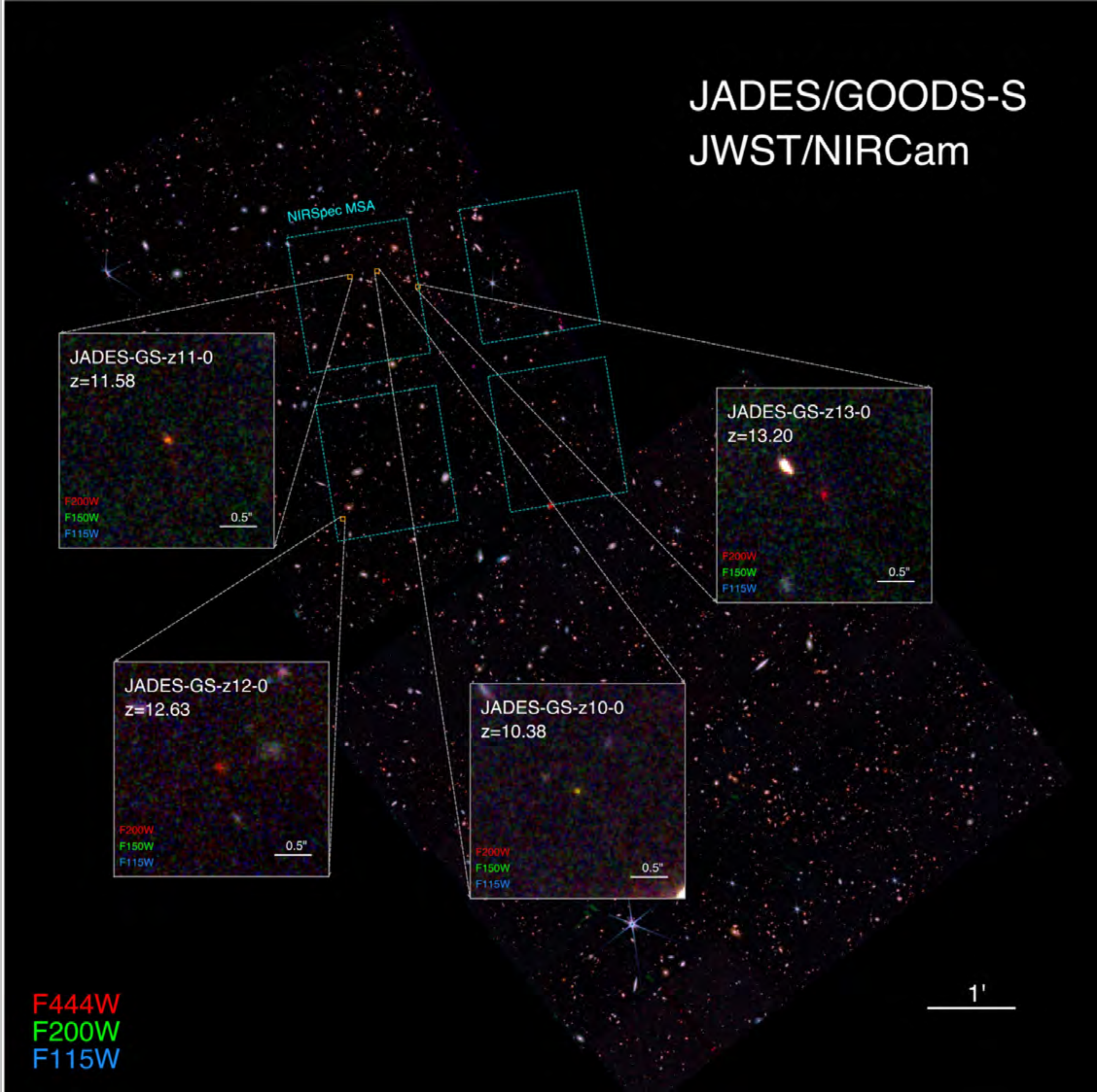

JWST (0.83 days)

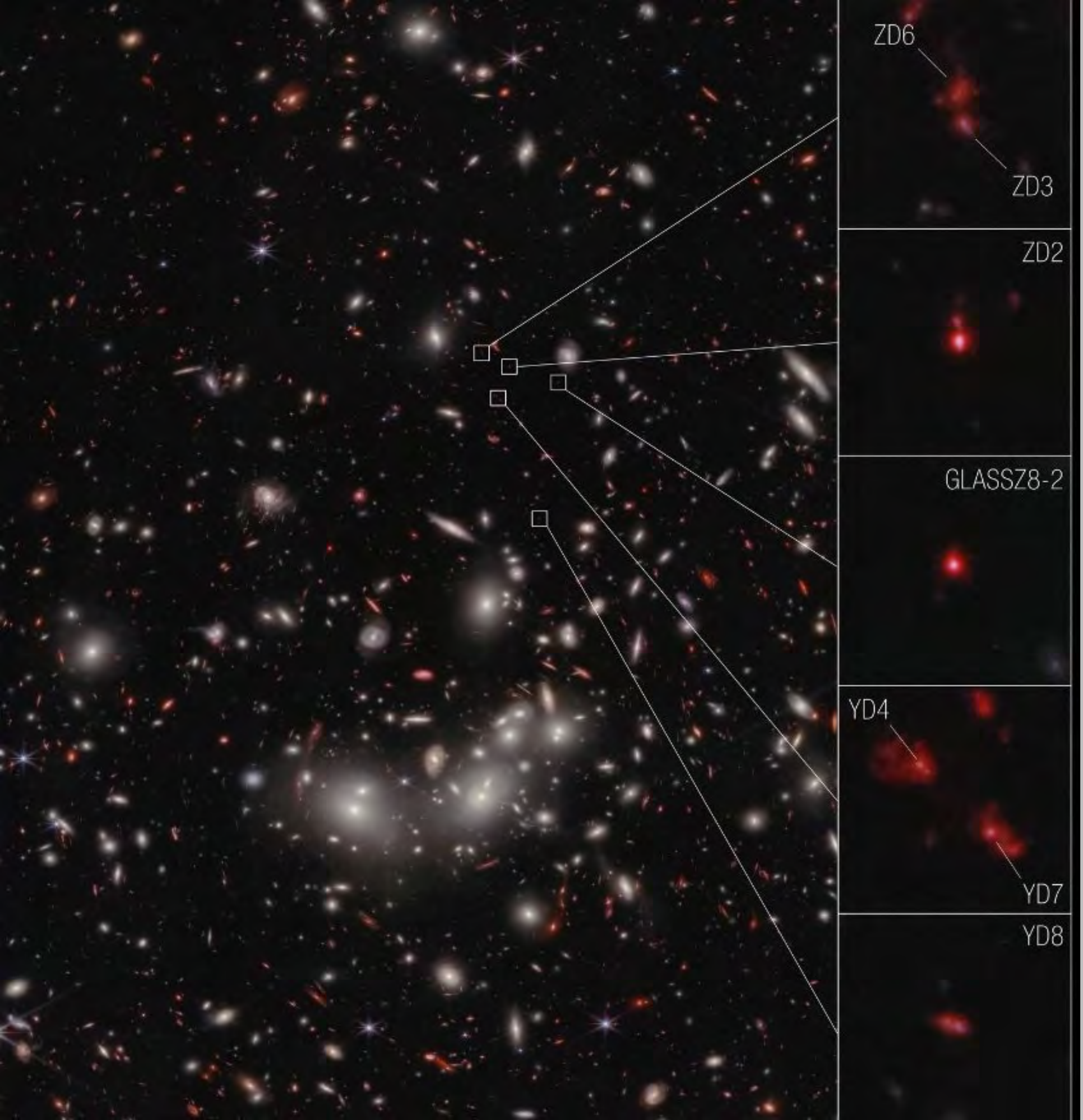
Galaxies at the highest (yet) redshift:

- Redshifts: $z \approx 10 - 13$
- Universe age: $300 - 450$ Myr
- Stellar mass: $\approx 10^8 - 10^9\ M_\odot$
- SFR: $\approx 0.2 - 5\ M_\odot/yr$
- Stellar ages: $\approx 15 - 70$ Myr

Robertson+23







High redshift proto-cluster:

- Proto-cluster behind foreground cluster
- Redshift $z = 7.9$
- Universe age: 650 Myr