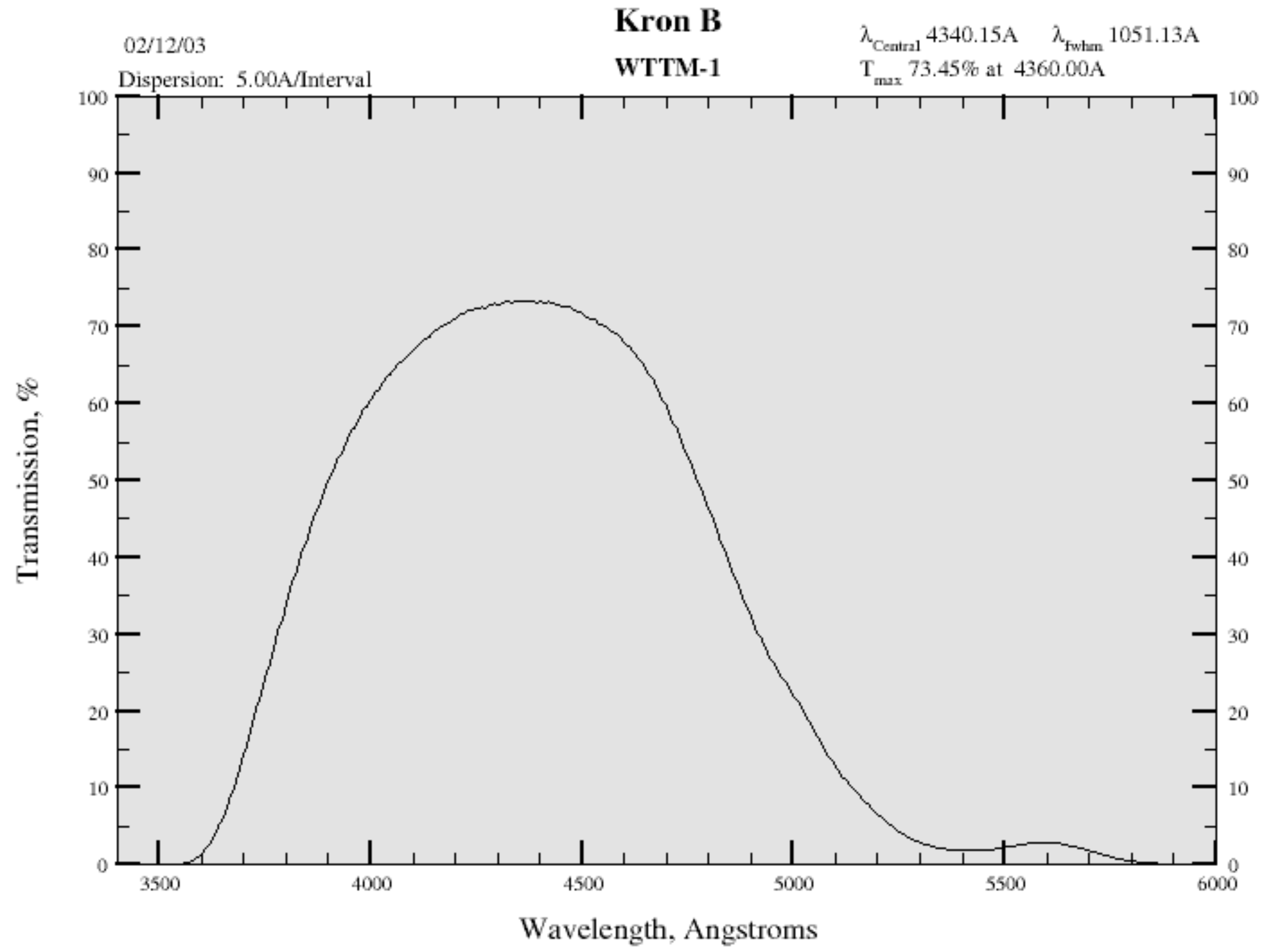
Filters and Magnitudes

Filter Transmission Curve

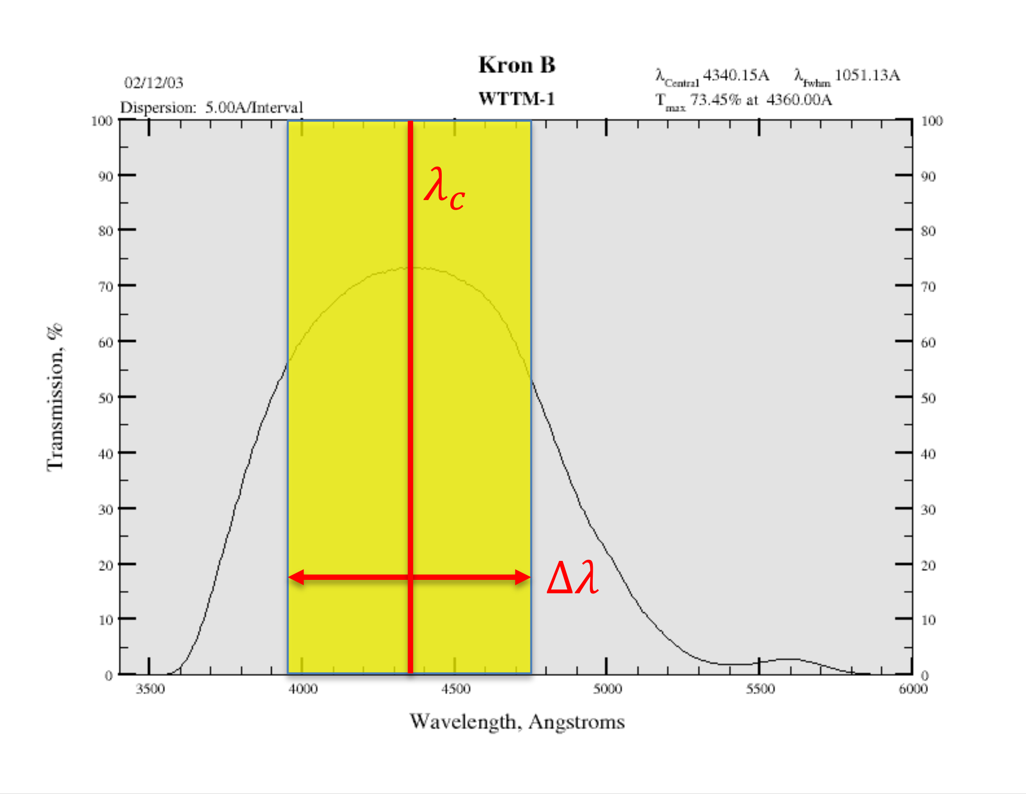


Filter specifications

Filters are not perfectly repeatable, due to manufacturing differences, environment differences, etc. One person’s B filter is slightly different from another’s.

Defining a filter (a reasonable attempt):

Transmission-weighted mean wavelength: $\lambda_c = \frac{\int \lambda T_\lambda d\lambda}{\int T_\lambda d\lambda}$



Filters don’t always look this regular!

Filter width:

- Δλ = FWHM (if transmission curve is reasonably symmetric)

or

- Δλ = width of the equivalent square filter at T=100% giving same throughput (i.e., area of yellow rectangle in the plot is the same as the integrated area underneath the curve....)

To standardize photometry, filter corrections are typically needed, and they depend on the color of what you are observing. So, for example, $m_B = m_{B,obs} + C(B-V)$ where C is called the “color term” of the individual filter.

Choosing filters

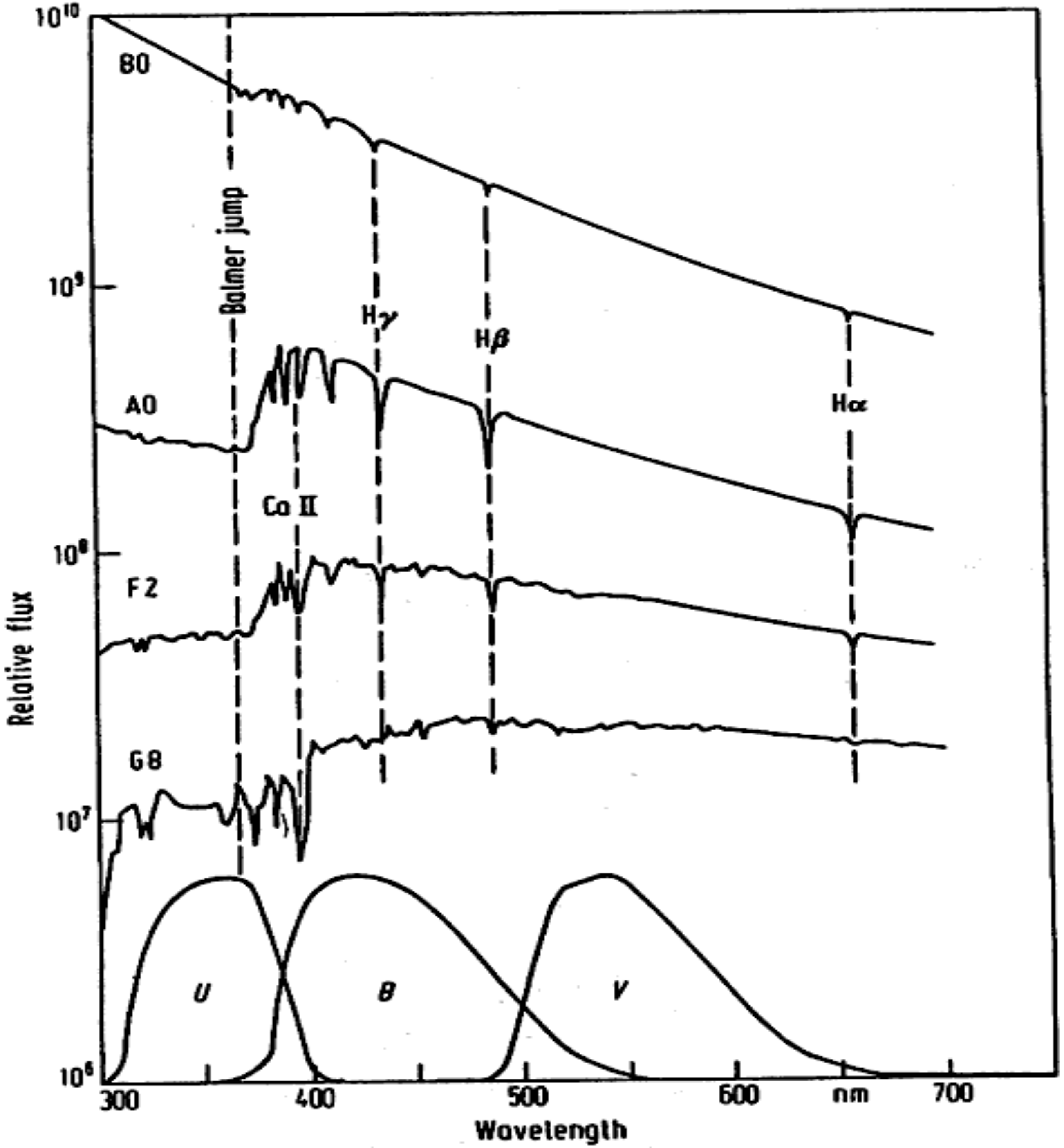
Trying to measure physical features of an astronomical spectrum.

For example, Johnson broadband filters. Δλ = 700 – 1000 Å

Colors measure spectral properties of stars.

B−*V*: slope of stellar continuum

U−*B*: strength of Balmer jump

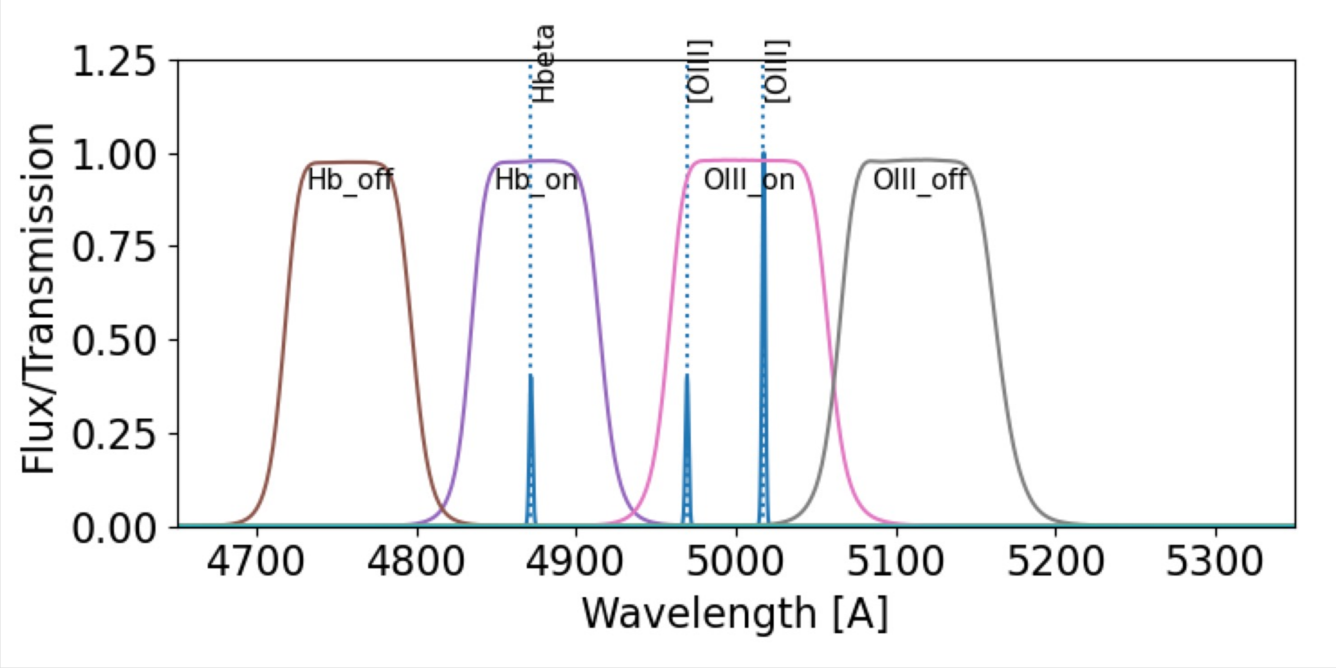


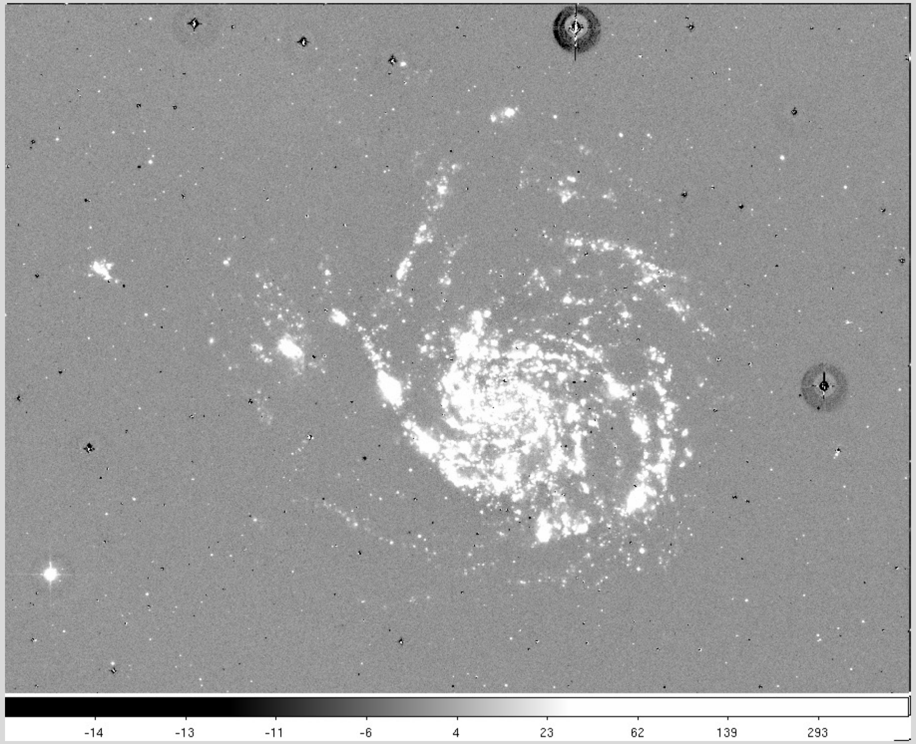
Choosing filters

Trying to measure physical features of an astronomical spectrum.

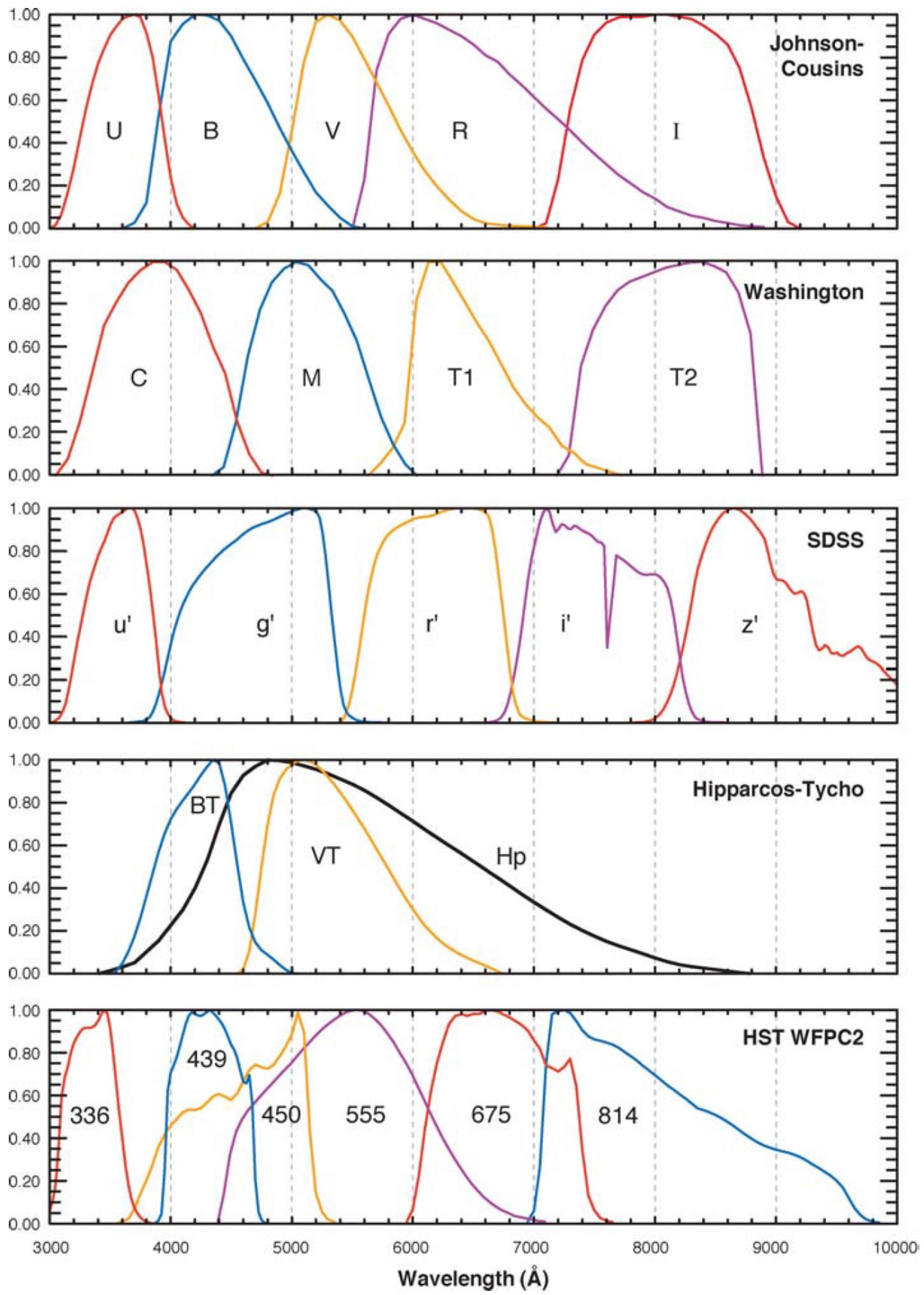
Or narrowband filters: measure the amount of flux in a particular emission line.

For example $H\beta, [OIII]$ filters,
Δλ = 80 − 100 Å





Common
Filter systems



Classic Johnson-Cousins filters

SDSS "ugriz" filters

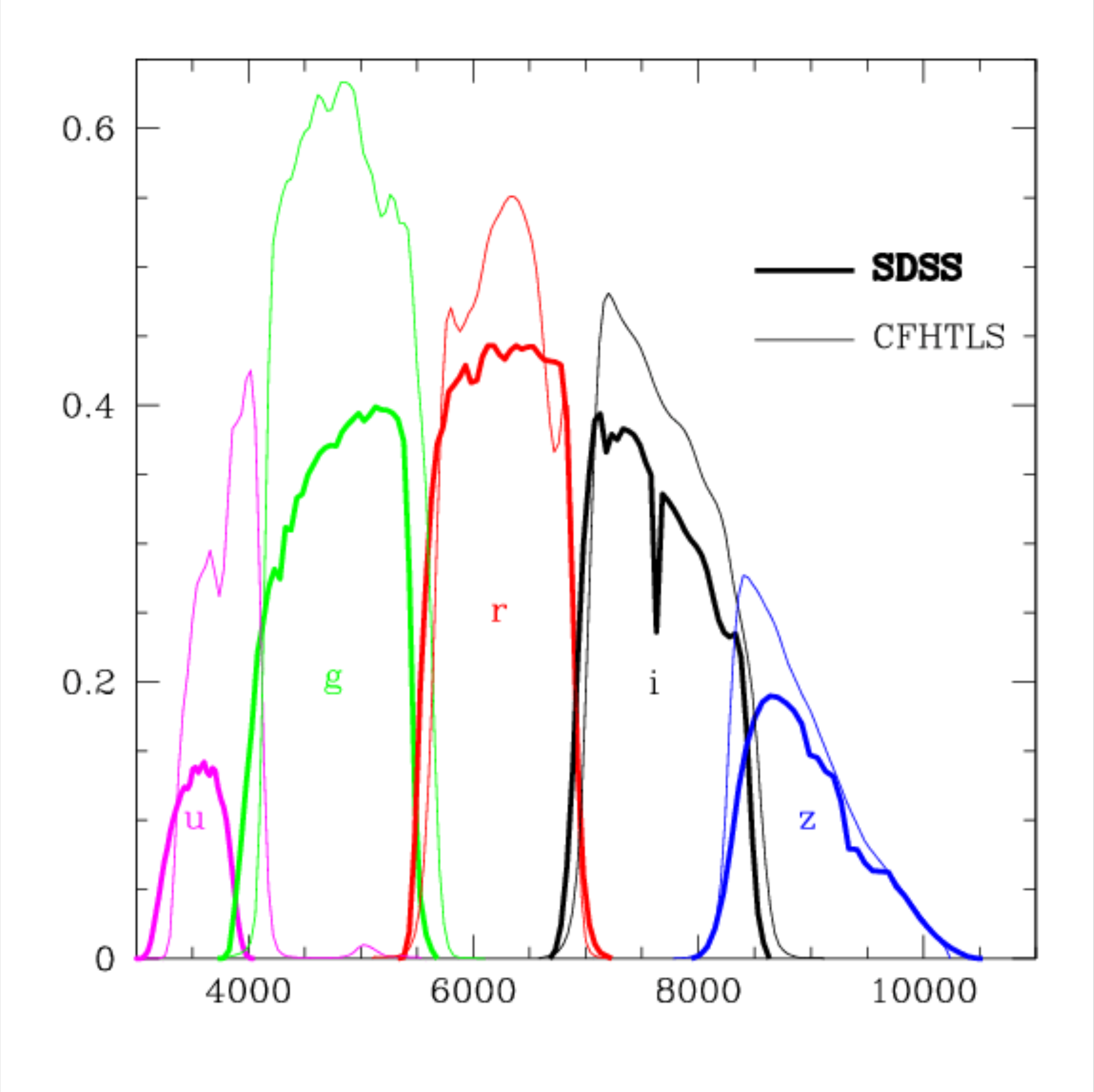
Hubble Space Telescope (older camera)

[Bessell ARAA 2005]

Filter system differences

example of “similar but different” filters:

SDSS ugriz vs CFHT ugriz



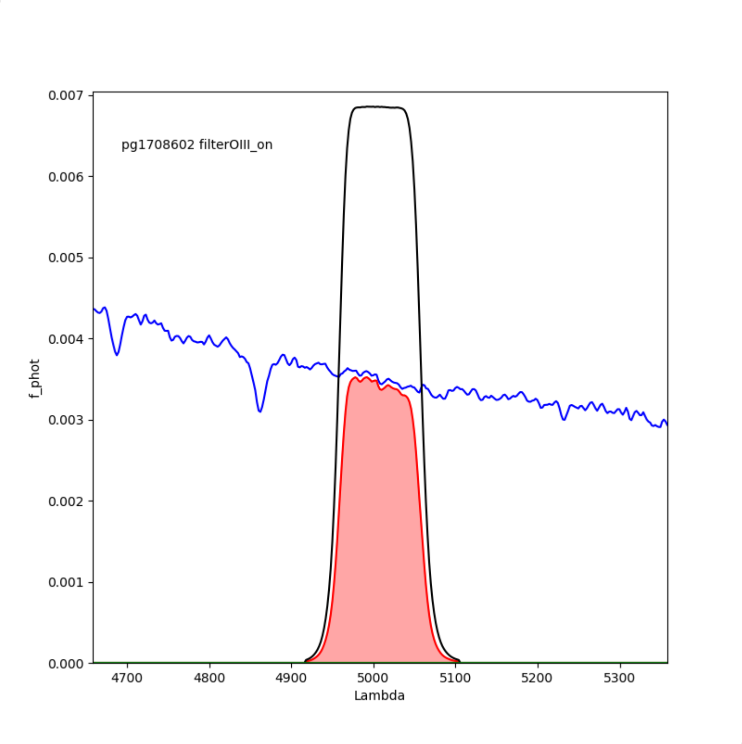
Flux through a filter

An astronomical object emits a spectrum given by f_λ (in erg/s/cm²/Å). This is referred to as spectral flux density (flux per wavelength).

Note: Flux density is often also written in terms of frequency: f_ν (in erg/s/cm²/Hz). Stay tuned for more about that....

The total flux (in erg/s/cm²) passing through the filter is given by

$$f = \int f_\lambda \times T_F(\lambda)\, d\lambda$$



f_λ: star spectrum

$T_F(\lambda)$: filter transmission (goes from 0.0 to 1.0)

$f_\lambda \times T_F(\lambda)$: spectrum through filter

$\int f_\lambda \times T_F(\lambda)\, d\lambda$: flux through filter

Magnitudes as a measure of flux

If fluxes (f) are in physical units (e.g., erg/s/cm^2), magnitudes of different objects ***measured in the same filter*** are related by

$$m_1 - m_2 = -2.5\log_{10}(f_1/f_2)$$

Magnitudes are defined relative to some standard flux or object.

- Δm = 1 mag ➔ factor of 2.512 in flux
- Δm = 5 mag ➔ factor of 100 in flux

Using differential calculus, if the magnitude uncertainties (σ) are small you can show that

$$\sigma_m = -1.086\left(\frac{\sigma_f}{f}\right) \approx \left(\frac{\sigma_f}{f}\right)$$

In other words, for small uncertainties the uncertainty in magnitudes is the fractional uncertainty in flux.

Magnitudes as a measure of distance

$$m - M = 5\log_{10}(d) - 5$$

m = apparent magnitude (measure of flux)
M = absolute magnitude (measure of luminosity)
$m - M$ = "**distance modulus**"
d = distance **in parsecs**

M87 at a distance of ≈ 16.5 Mpc has a distance modulus of

$$m - M = 5\log(16.5\times10^6) - 5 = 31.1 \text{ mags}$$

Again using differential calculus, if the uncertainties (σ) are small you can show that

$$\frac{\sigma_d}{d} \approx 0.5\sigma_{(m-M)}$$

In other words, the fractional uncertainty in distance is about half the uncertainty in the distance modulus.

Surface Brightness

If magnitude is defined by $m = -2.5 \log f + C$, we can define **surface brightness** ($\boldsymbol{\mu}$) as flux (f) per unit angular area (A) on the sky:

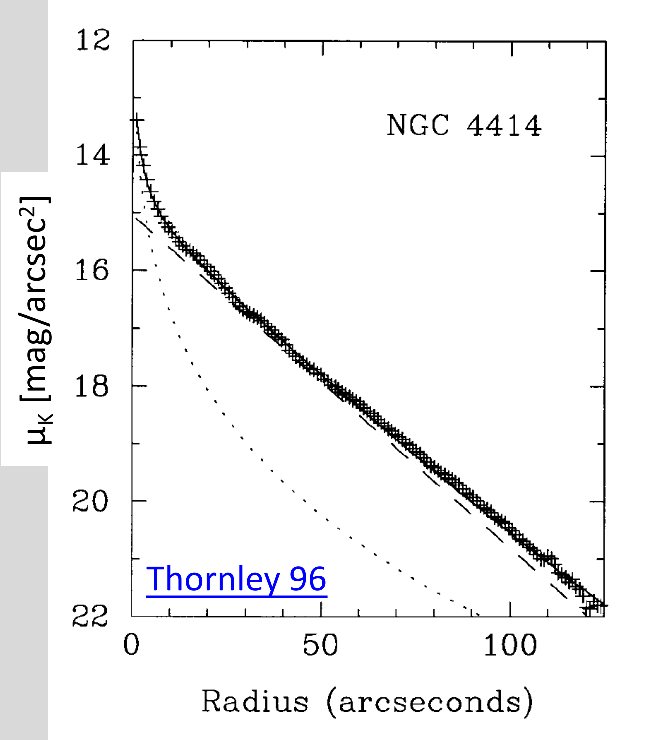
$\mu = -2.5 \log(f/A) + C$

or
$\mu = -2.5 \log f + 2.5 \log A + C$

So
$\mu = m + 2.5 \log A$

if angular area is measured in arcsec^2, then surface brightness (μ) is given in $\mathrm{mag/arcsec}^2$

However, units not withstanding, surface brightnesses (just like magnitudes) are not additive. *Surface brightness is **not** magnitude divided by area!*



Surface Brightness

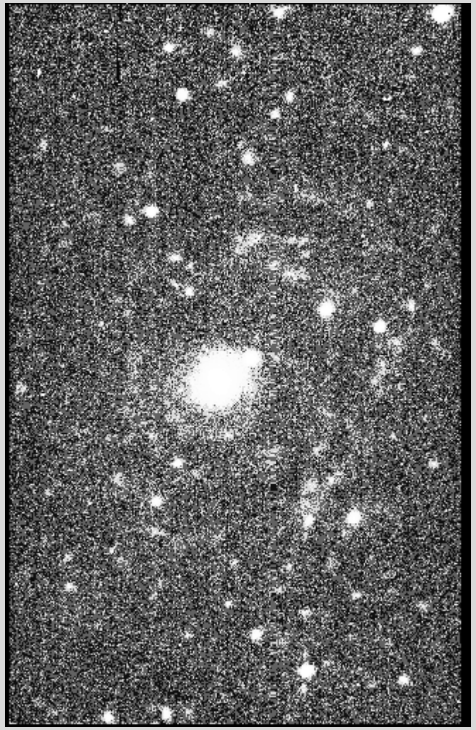
Surface brightness is **distance independent**, an intrinsic property of the object being studied (at least until you get to cosmological distances).

Therefore an *observable surface brightness* (in mag/arcsec^2) corresponds to an *intrinsic luminosity (surface) density.*

For example, μ_B = 27.0 mag/arcsec^2 corresponds to ≈ 1 $L_{B,\odot}/pc^2$.

M101

Malin 1



Colors

Magnitudes are always defined relative to some reference object/value, so the zeropoint C_λ depends on the reference system.

Color $\equiv m_{\lambda_1} - m_{\lambda_2}$, so for example $B - V = m_B - m_V$

If $m_\lambda = -2.5 \log f_\lambda + C_\lambda$, then

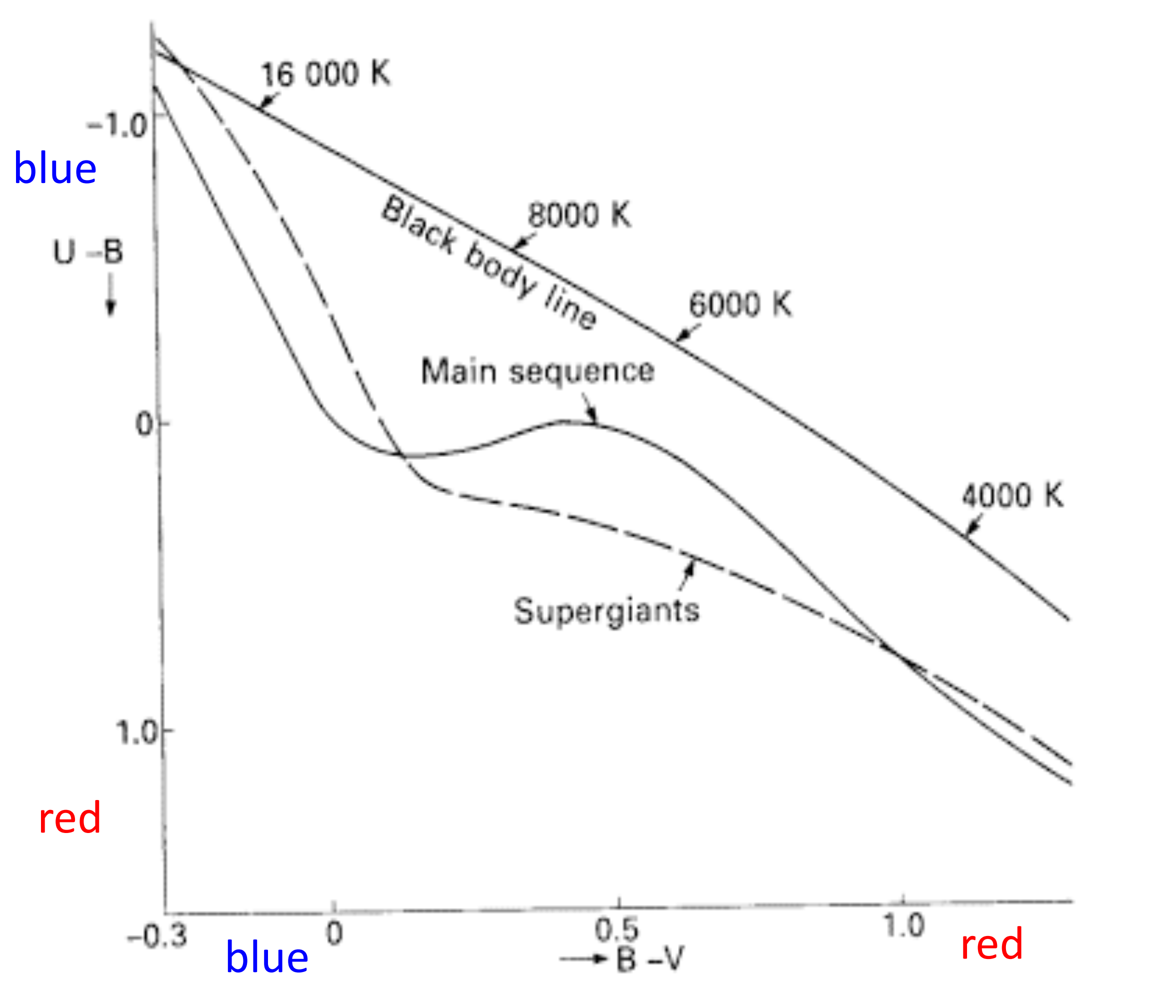
$$\text{Color} \equiv (-2.5 \log f_{\lambda_1} + C_{\lambda_1}) - (-2.5 \log f_{\lambda_2} + C_{\lambda_2}),$$

or

$$\text{Color} \equiv -2.5 \log(f_{\lambda_1}/f_{\lambda_2}) + (C_{\lambda_1} - C_{\lambda_2})$$

Important points:

- Like magnitudes, colors are measured relative to some reference object (the origin of the C_λ term)
- Convention is to always list the bluer filter first. So B−V, not V−B.
- This means that smaller (and more negative) numbers are bluer colors.



UBV color-color plot for stars

Magnitude Systems *(or "what's the zeropoint?")*

Don't confuse magnitude systems with filter systems! – M Bershady

$$m_\lambda = -2.5 \log f + C_\lambda$$

Conceptually, the zeropoint (C) can either be based on physical units or on a reference star.
See Bessell (ARAA) 05 for review.

The Vega System

By definition, Vega (α Lyr): m = 0.00 *at all wavelengths*:

$$m_B = m_V = m_R = m_I \equiv 0.0$$

Therefore Vega has a color of 0.00 in all colors *by definition*:

$$B - V = V - I = I - R = 0.0$$

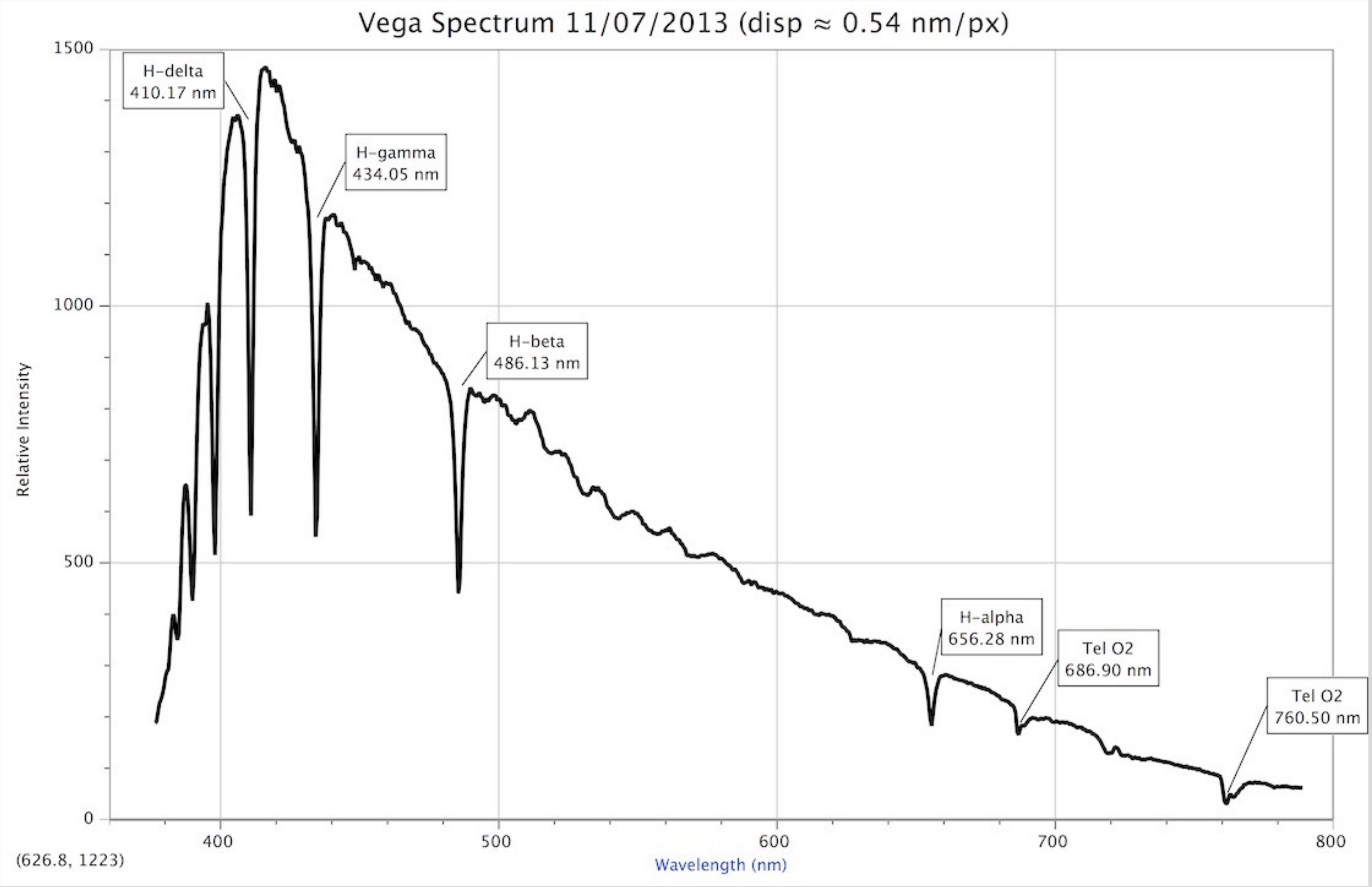
Therefore, in the Vega system, a color of 0.0 is ***NOT*** the same as equal flux at all wavelengths (a so-called "flat spectrum").

Magnitudes measure brightness ***relative to Vega*** and colors measure colors ***relative to Vega***.

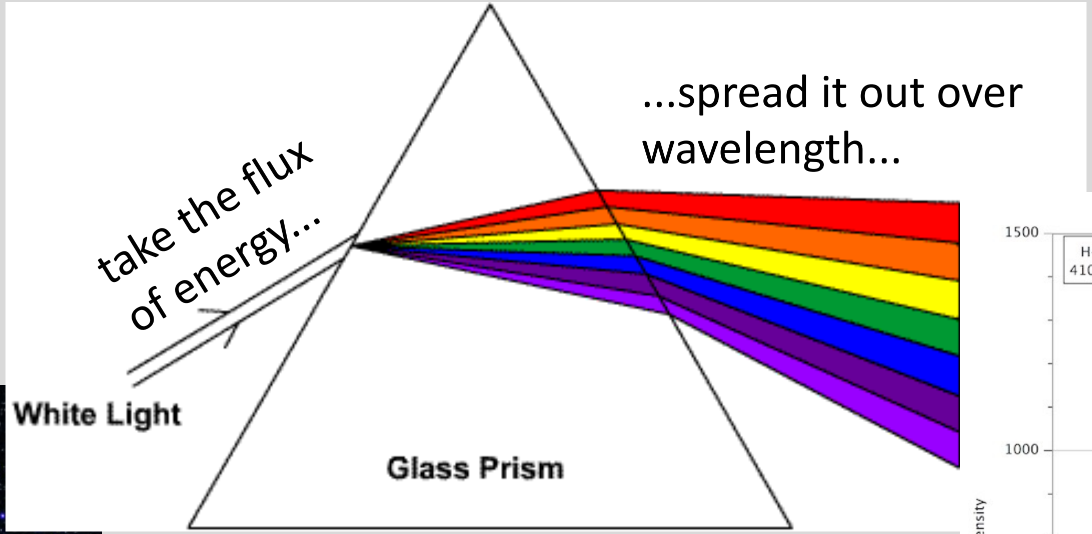
Vega is a very blue star!

Vega spectrum

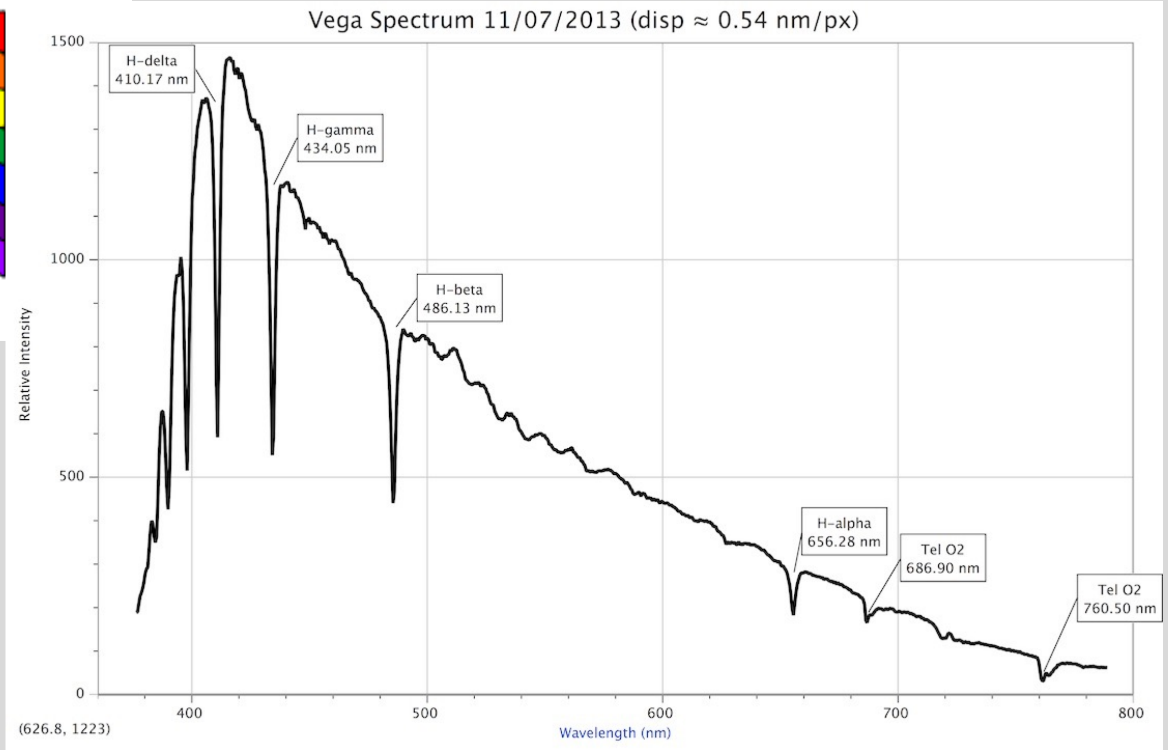
Courtesy KSU Astronomy



Physical Units: Flux and Flux Density



...to create a spectrum. Flux density is the intensity of the spectrum



Flux: Energy/area/time
Units: erg/s/cm^2
(where cm^2 refers to the area of your light collector)

Flux density: Energy/area/time/wavelength
Units: erg/s/cm^2/Angstrom

Magnitude Systems: the AB and STMAG systems

We can define the **monochromatic flux density** as

f_ν = Energy/area/time/frequency = erg/s/cm²/Hz

(1 Jansky = 10^{-23} erg/s/cm²/Hz)

or

f_λ = Energy/area/time/wavelength = erg/s/cm²/Å

Relating f_ν and f_λ

$$f_\nu d\nu = -f_\lambda d\lambda$$

or (since $\nu = hc/\lambda$)

$$f_\nu = \left(\frac{\lambda^2}{c}\right) f_\lambda$$

So there are two monochromatic magnitude systems where the zeropoint is in physical units of flux density:

AB system	STMAG system
$m_{AB} = -2.5 \log f_\nu - 48.6$	$m_{ST} = -2.5 \log f_\lambda - 21.1$
f_ν measured in erg/s/cm²/Hz	f_λ measured in erg/s/cm²/Å
color = 0 means constant f_ν	color = 0 means constant f_λ

Important points:

- Zeropoints are chosen so that in V band (≈5500Å), Vega has $m_{AB} \approx m_{ST} \approx 0.0$
- AB system more common than STMAG; SDSS *ugriz* mags are AB mags
- Constant f_ν is not the same as constant f_λ

Photometric Systems: Magnitude Zeropoints vs Flux Zeropoints

Think about the basic magnitude definition: $m = -2.5 \log f + C$

Written that way, C is a **magnitude zeropoint**, the magnitude of an object with $f = 1$ (in the appropriate units).

A different way of writing it would be: $m = -2.5 \log(f/f_0)$, where f_0 is the **flux zeropoint**, i.e., the flux of a zeroth magnitude object.

The two are related mathematically by $C = 2.5 \log f_0$

- In the AB system, the magnitude zeropoint is **the same at all wavelengths**: $\mathrm{C} = -48.6$. From this you can work out the flux zeropoint in erg/s/cm^2/Hz, and then convert that into Janskys.

- In the Vega system, the brightness of an object is measured relative to the brightness of Vega at each wavelength, **the zeropoints change with wavelength.** For example:

B (Vega)	V (Vega)
$f_0 = 4260$ Jy	$f_0 = 3640$ Jy

Remember: 1 Jy = 10^{-23} erg/s/cm^2/Hz

Handy table of zeropoints for different magnitude systems (Paul Martini, OSU)

Photometric Systems: Colors

Remember that a color is the difference between magnitudes at two wavelengths, for example B and V:

$$B - V = m_B - m_V = (-2.5\log(f_B) + C_B) - (-2.5\log(f_V) + C_V)$$

or equivalently

$$B - V = m_B - m_V = \left(-2.5\log(f_B/f_{0,B})\right) - \left(-2.5\log((f_V/f_{0,V})\right)$$

depending on whether you are using magnitude zeropoints or flux zeropoints.

Because these zeropoints are different in different magnitude systems (say Vega vs AB), a star will have a different color in different magnitude systems.

- In the Vega magnitude system, Vega has a color of $B - V = 0.00$, by definition.
- In the AB system, Vega has color of $B - V = -0.07$, its is slightly bluer than an object with constant f_ν

Moral of the story: always check to see what magnitude system is being used: Vega, AB, or STMAG.

Worked Example: Vega in different units

For Vega, the monochromatic flux density at 5492Å is

$f_\lambda = 3.63 \times 10^{-9}$ erg/s/cm^2/Å

which can also be written in terms of frequency:

$f_\nu = (\lambda^2/c)f_\lambda = 3.65 \times 10^{-20}$ erg/s/cm^2/Hz = 3650 Jy

careful with units on this step: Since f_λ was in "per Å" and f_ν is in "per Hz", λ and c should be in Å and Å/s respectively!

or AB magnitudes:

$m_{AB} = -2.5\log(f_\nu) - 48.6 = -0.006$

to convert to photon flux, divide by f_λ by the photon energy (hc/λ):

photon flux ≈ 1000 photons/s/cm^2/Å

and if the V filter has a width of ~ 900 Å, the total photon flux through a V filter bandpass is about 900,000 photons/s/cm^2.

Remember: these are all "top of the atmosphere" values, i.e., airmass X=0.

why do we care about photon flux? detectors count the number of photons received, not the amount of energy received!