ASTR 222 Homework #3: Due Mar 7 2025

Population Synthesis (20 points)

We are going to make galaxies by mixing together different types of stars, shown in the table below. For each type of star, describe in words what kind of star it is: what evolutionary stage it is in, what kind of lifetime it has, what limits (if any) you can place on its age. Then calculate its stellar mass-to-light ratio $(\mathcal{M}/L)_*$.

	Star 1	Star 2	Star 3	Star 4
Spectral Type	A2V	G2V	K5V	K2III
Absolute Mag (M_V)	1.3	4.8	7.35	0.5
$B-V$ color	0.05	0.65	1.15	1.16
Mass ($\mathcal{M}_\odot$)	2	1	0.67	1.1
Mass-to-Light Ratio $(\mathcal{M}/L)_*$	*(fill this line in for each star!)*			

(Remember that mass-to-light units are solar units, so the Sun has a mass-to-light ratio of $(\mathcal{M}/L)_* = 1\ \mathcal{M}_\odot/L_\odot$. And also remember that the Sun has a V-band absolute magnitude of $M_V = +4.8$ and a color of $B-V = 0.65$.)

Now let's build some galaxies. The galaxies should each have a total V-band luminosity of $L_V = 10^{10}\ L_\odot$. The fraction of the total V-band light each star contributes to each galaxy is given in the table below. Calculate the total ("integrated") $B-V$ color and V-band stellar mass-to-light ratio $(\mathcal{M}/L)_*$, as well as the fraction of each star by number for each model galaxy. Show your work, by explaining a calculation for Galaxy 1 in exquisite detail.

	Fraction of V-band light from each star			
	Star 1	Star 2	Star 3	Star 4
Galaxy 1	15%	40%	25%	20%
Galaxy 2	30%	0%	0%	70%
Galaxy 3	45%	25%	20%	10%
Galaxy 4	0%	30%	70%	0%
Galaxy 5	0%	30%	50%	20%

Now, a "typical" color for a spiral galaxy like the Milky Way is $B - V = 0.7$, an elliptical might have a color of $B - V = 1.0$, and a starburst galaxy (that is forming stars at a furious rate) might have $B - V = 0.4$.

Which of these galaxies is a good match for an elliptical, which for a spiral, and which for a starburst? Which two galaxies don't make sense? Argue your answer both from integrated colors and from the mix of stellar types.

Surface brightness (10 points)

If a galaxy is observed face on (with no dust) and has a luminosity density equivalent to one solar-type star per square parsec, show that the surface brightness in magnitudes per square arcseconds is $\mu_V = 26.37$ mag/arcsec^2 in the V band.

Hint: Imagine putting the Sun in a box that is 1pc on a side; that box then has a luminosity density of 1 $L_\odot\ pc^{-2}$. Now move that box to a distance where the box has an apparent size of 1 arcsecond on a side. What is the apparent magnitude of the Sun? So what is the observed surface brightness of the box?

From this, we can express the relationship between surface brightness and luminosity density as $\mu_V = 26.37 - 2.5\log(I_V)$, where I_V is the V-band luminosity density of the galaxy in solar luminosities per square parsec.

If a galaxy has a central surface brightness of $\mu_V = 21.0$ mag/arcsec^2, what is the luminosity density in the center of the galaxy? We are now discovering really faint "ultradiffuse galaxies" that have central surface brightnesses of $\mu_V = 27.0$ mag/arcsec^2 -- what does that correspond to in terms of luminosity density?

Disk Galaxies: Luminosity, Rotation Curves and Dark Matter (20 points)

If a disk galaxy has an exponential surface brightness profile $I = I_0 e^{-(R/h)}$, where h is the scale length and I_0 is the central luminosity density (in $L_\odot$/pc^2), calculate:

1. the total luminosity of the galaxy (in terms of I_0 and h).
2. the half-light radius of the galaxy (in terms of h)
3. the radius containing 98% of the total light (in terms of h).

If the galaxy has a constant stellar mass-to-light ratio $(\mathcal{M}/L)_*$, derive an ***analytic expression*** for what the rotation curve of the galaxy should look like (ignore the bulge and halo of the Galaxy for this calculation). For your expression for the rotation curve, I want something that looks like $V_c(R) = f(I_0,\ (\mathcal{M}/L)_*, h, R)$.

Okay, now if the galaxy has a central surface brightness $\mu_0 = 19.2$ mag/arcsec^2, $(\mathcal{M}/L)_* = 1.0\ \mathcal{M}_\odot/L_\odot$, and $h = 3.5$ kpc, plot what the rotation curve of the galaxy should look like over the range $R = 0$ to 30 kpc. *(Hint: if it looks flat, you screwed up.)*

Now, the observed rotation curve is roughly flat; at $R = 30$ kpc, the circular velocity is still $V_c \approx 220$ km/s. What is the mass needed to give this circular velocity? How much disk mass is there inside $R = 30$ kpc? So how much dark matter do we need? So inside $R = 30$ kpc, what percentage of the Galaxy's mass is in dark matter?

Some studies suggest that the rotation curve of the Galaxy remains flat out to 150 kpc (or further!). If so, what fraction of the Galaxy's mass (inside 150 kpc) is in the form of dark matter?

Donkey Dark Matter (15 points)

One of my favorite ideas about the infamous "dark matter" surrounding galaxies is that it is made up of a population of free floating space donkeys (FFSDs). FFSDs would not radiate in the optical (have you ever seen a donkey shine?) but would emit light in the infrared (since they would have little heat generators in the donkey space suits to keep them warm). So let's see if we can rule out this model.

Say the dark matter halo of a bright spiral galaxy like the Milky Way has a mass of $10^{12}\ \mathcal{M}_\odot$. If it was made of FFSDs, what would the bolometric (total) luminosity of the dark matter halo be? (Assume FFSD are blackbody radiators.) What would the peak wavelength of this light be? Compare this to the bolometric luminosity of the Milky Way stars ($\approx 5 \times 10^{10} L_\odot$). How many times brighter or fainter are the FFSDs than the stars? Do you think we'd be able to detect them?