# ASTR 222 Homework #4

### 1. Working with the Tully-Fisher Relationship (20 points)

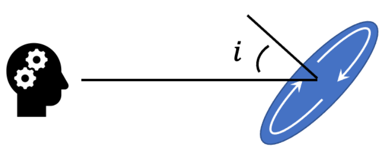
The Tully Fisher Relationship for spiral galaxies can be expressed by $M = a \log\left(\frac{V_c}{300\ km/s}\right) + b$, where $M$ is the absolute magnitude of the galaxy (in some filter band), $V_c$ is the galaxy's rotational velocity, and $a$ and $b$ are the calibrated parameters of the relationship. Expressed this way, $b$ is actually the absolute magnitude of a galaxy that rotates at 300 km/s. Note also that since absolute magnitudes depend on which band you observe in (say, $B$ or $V$), the fitted parameters $a$ and $b$ will also differ depending on which band you are working in.

In general, unless we know the distances of galaxies, we can't make a Tully-Fisher plot, since we don't know absolute magnitudes without knowing distances. But we can be crafty and realize that if we look at galaxies within a single galaxy cluster we can plot **apparent** magnitude against $\log\left(\frac{V_c}{300\ km/s}\right)$ and still get the proper slope ($a$). Explain why this is.

So now here is a Tully Fisher dataset for galaxies in the Virgo Cluster (from Pierce & Tully 88). The dataset has the following information:

- the NGC catalog number of each galaxy,
- the **apparent magnitude** of the galaxies in blue ($B$), red ($R$), and near-infrared ($I$) filters.
- the **inclination** of the galaxy to our line of sight (in degrees, where $inc = 0°$ means the galaxy is seen face-on and $inc = 90°$ means the galaxy is seen edge-on; see the sketch below).
- the **observed rotation speed** ($V_{c,obs}$) of the galaxies in km/s.

First, we need to make an inclination correction to convert the *observed* rotation speed ($V_{c,obs}$) to the *true* rotation speed ($V_c$) of the galaxy. Use the sketch below to work out a trigonometric function for the correction (show your work), then apply this correction to the observed rotation speeds to get the true rotation speeds.




Now make a Tully-Fisher plot (y-axis: apparent magnitude $m$, x-axis: $\log\left(\frac{V_c}{300\ km/s}\right)$ ) for each of the $B$, $R$ and $I$ bands, and each case, fit a line to the observed Tully-Fisher relationship: $m = a \log\left(\frac{V_c}{300\ km/s}\right) + b'$, where $b'$ is different from $b$, since we are using apparent magnitudes instead of absolute magnitudes. For the fit in each band, give the slope ($a$) and its uncertainty, the intercept ($b'$) and its uncertainty, and the scatter in the fit. Recent studies have shown that the scatter in the Tully-Fisher relationship is fairly small, about 0.2—0.3 magnitudes. Your measurements will come out *higher* than that, though. Give a few reasons why the scatter you derive is so large.

Also give physical arguments about which band would best define the Tully-Fisher relationship. Think both about stellar populations and dust.

Now we need to calibrate the Tully-Fisher relationship. We want to know how absolute magnitude depends on circular velocity, which means we need to know a distance to the Virgo cluster. Using the Hubble Space Telescope, we can use Cepheid variables in the spiral galaxy M100, also known as NGC 4321. Here are the reduced light curves (from Freedman+ 94). These are plots (one for each detected Cepheid) of apparent magnitude on the y-axis and "phase" ($\phi$) on the x-axis. ("Phase" means time relative to the period of the variable star, so $\phi = 0.5$ means halfway through the period). The period (in days) of each are given above each plot.

From the plots, estimate the average apparent magnitude of each Cepheid (an eyeball estimate is fine!), then use the Cepheid period-luminosity relationship ($M = -2.43 \log P - 1.62$, where $P$ is the period in days) to calculate a distance estimate to M100 for each Cepheid. Average all those estimates together to give your best estimate for the M100 distance. Also give a statistical uncertainty to your distance.

Now use this distance to calibrate the Tully-Fisher relationship. You had a T-F relationship that connected apparent magnitude ($m$) with $\log V_{c,true}$, and now you know the distance to Virgo to turn apparent magnitude ($m$) into absolute magnitude ($M$), so you can rewrite your T-F relationship now in terms of absolute magnitude: $= a \log\left(\frac{V_c}{300\ km/s}\right) + b$. What are $a$ and $b$, and their uncertainties ($\sigma_a$ and $\sigma_b$)? Remember now that the uncertainty in $b$ should incorporate your uncertainty in both the fit and in the Virgo distance estimate.

What do you feel are the main sources of uncertainty, both random (i.e., the error in fitting and calculations) and systematic (the uncertainty due to the over conceptual method), in your derivation of the full calibrated Tully-Fisher relationship?

Finally, imagine you are looking at a spiral galaxy in the Coma cluster. It has an I-band **apparent** magnitude of $m_I = 13.5$, an **observed** rotation speed of $V_{c,obs} = 180$ km/s, and an inclination of $i = 65°$ . What is the galaxy's absolute magnitude, and its uncertainties (propagate the uncertainties in the T-F fit ($\sigma_a$ and $\sigma_b$) to get the uncertainty on the absolute magnitude). From this absolute magnitude estimate, work out the distance to Coma and the uncertainty in that distance. To get that uncertainty in distance, use the approximation that the relative uncertainty in distance ($\sigma_d/d$) is roughly half the magnitude uncertainty in the distance modulus.

**2. The Fundamental Plane (10 points)**

The Fundamental Plane for elliptical galaxies connects the luminosity density ($I$, in $\mathrm{L}_\odot\ \mathrm{pc}^{-2}$), velocity dispersion ($\sigma$, in km/s), and effective radius ($r_e$, in pc) via the relationship $r_e \sim \sigma^x I^y$, where the parameters of the relationship are given by $x = 1.24, y = -0.82$. Let's work out what this means in terms of how the mass-to-light ratio of elliptical galaxies depends on luminosity.

We'll do this conceptually, without worrying too much about constants, like we did when we worked out a physical interpretation of the Tully-Fisher relationship (see slide 49 in the Galaxies notes online). So just worry about variables like size, mass, luminosity, etc – not $G$'s or $\pi$'s or the like….

First, ignore the Fundamental Plane for a minute and write down an expression that shows how mass scales with velocity and size. Then write down an expression for how luminosity scales with luminosity density and size. Combine those two expressions to show how total mass-to-light ratio ($\mathcal{M}/L$) depends on $\sigma, I,$ and $r_e$.

Now combine the Fundamental Plane relationship with the relationship you just worked out for total mass-to-light ratio to show that $(\mathcal{M}/L) \sim L^{\alpha} I^{\beta}$, and work out the values of the parameters $\alpha$ and $\beta$. The parameter $\beta$ should turn out to be close to zero (i.e., $\beta \lesssim 0.05$ or so), meaning that mass-to-light ratio is pretty insensitive to the luminosity density, such that to a very good approximation the mass-to-light ratio of an elliptical galaxy depends ***primarily*** on luminosity. Remember here we are talking about the ***total*** mass-to-light ratio, meaning the mass of ***everything*** in the galaxy (stars + dark matter) divided by the luminosity.

Finally, make some astro-educated arguments about why the total mass to light ratio of an elliptical galaxy might be higher for more luminous ellipticals. Think about both stellar populations and dark matter.

**3. The Mass of the Coma Cluster (20 points)**

In this problem we are going to estimate the total mass of the Coma cluster of galaxies. For this problem, adopt a Hubble constant of $H_0 = 72$ km/s/Mpc.

First, go look at an image of the Coma cluster online, so that you know what you are studying.

Now [here is a dataset of galaxies in a 6x6 degree field around the Coma cluster](#) (from [Doi+ 95](#)). It contains

- name: galaxy ID number
- x & y: angular position relative to the cluster center of Coma (defined by NGC 4886), measured in arcminutes
- cz: the observed radial velocity of the galaxy in km/s. cz=0 means no measurement.
- Bt: the blue ($B$) magnitude of each galaxy

First make an x,y plot of the galaxy distribution. (make sure the axes on your plot have a square aspect ratio; use `set_aspect('equal')` when making your plot!) See -- it's a cluster!

Now make a histogram of the radial velocity of all the galaxies (use bins with width of 250 km/s, and don't include galaxies without velocity measurements when making your histogram!). Explain how this plot would help you decide which galaxies actually were part of the Coma cluster?

Looking only at galaxies with $4{,}000 < cz < 10{,}000$ km/s (explain why this cut is important), calculate the mean velocity and velocity dispersion ($\sigma_v$) of the galaxy sample. From your data, how far away is Coma (in Mpc)?

Now, create a subsample of Coma cluster galaxies that meet the following criteria:
Your sample should consist of galaxies that meet the following criteria:

- Have a redshift within range $\pm 2\sigma_v$ of your Coma redshift, and
- Have a magnitude fainter than $m_B = 13$.

***And from now on in the problem, we will only be working with the galaxies in your sample.***
For the galaxies in your sample, calculate their distance from the center of the cluster (in arcminutes), and remember the cluster center is at (X,Y) = (0,0).

Then estimate the total blue luminosity (in solar luminosities) for all the galaxies in your sample, combined. Remember, *don't add magnitudes!* Calculate the luminosity of each galaxy individually, then add the luminosities of all the galaxies together. For reference, the absolute blue magnitude of the Sun is $M_{B,\odot} = +5.5$.

Next figure out the radius (in arcminutes) which contains roughly half the total blue luminosity of your sample (it doesn't have to be an exact solution, but you should get it to an accuracy of +/- 20 arcminutes or so). This is called the half-light radius, which is our estimate of the size of the Coma cluster. What is the half-light radius of Coma in Mpc?

Finally calculate the total mass of Coma (in solar masses), using the following expression for cluster mass: $\mathcal{M}_{tot} = 5\langle R\rangle\sigma_v^2/G$. Use your half-light radius (in parsecs!) as your estimate of $\langle R\rangle$, the characteristic size of the cluster.

Now the total luminosity of all the galaxies in Coma (not just the ones in your sample) is probably something like $L_B \approx 3 \times 10^{13}\ L_\odot$ or so. (This should be roughly 10 times the total luminosity of your sample, since a lot of the Coma galaxies do not have redshifts in that dataset.) Use that luminosity for the total stellar luminosity of Coma and calculate the following things:

- If the stars in the Coma cluster galaxies have an average ***stellar*** mass-to-light ratio of $(\mathcal{M}/L)_* = 3\ \mathcal{M}_\odot/L_{B,\odot}$, what is the total mass of stars in the Coma cluster?

- If the galaxies in the Coma cluster have an average ***total*** mass-to-light ratio of $(\mathcal{M}/L)_{tot} = 20\ \mathcal{M}_\odot/L_{B,\odot}$, what is the total galaxy mass of Coma (this will consist of stellar mass and the mass of any dark matter and gas which is contained inside galaxies).

- X-ray measurements indicate that the Coma Cluster has a hot gas mass of $\approx 3 \times 10^{14}\ \mathcal{M}_\odot$. What fraction of Coma is dark matter unassociated with galaxies (i.e., is dark matter distributed smoothly throughout the cluster)?