ASTR327 – HW #1

Due February 10, 2000

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1. For the spherical logarithmic potential $\Phi(r) = \frac{1}{2}v_0^2 \ln(r^2)$,

- Calculate the rotation curve $v_c(r)$.
- Compute the escape velocity at $r = 1$. In class, we said that $v_e = \sqrt{2|\Phi(r)|}$. We the potential defined as it is in this problem, that equation won't work. Why not?
- Compute the mass density of the system from Poisson's equation.
- Find apocenter and pericenter for an orbit with $L = 0$ and arbitrary E.
- Find the radius and L of the circular orbit with $E = 0$.
- Show that the L in the previous point is the maximum L possible at $E = 0$.
- Calculate the epicyclic frequency κ at $r = 1$.

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2. Using the definition of Oort's constants given in class, show that

- $A - B = \Omega_0$
- $\kappa_0^2 = -4B(A - B)$

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3. On the web site are two data files holding rotation curve data for the spiral galaxies NGC 2403 and UGC 128. Other important properties of these galaxies are:

	UGC 128	NGC 2403
Distance	60 Mpc	3.3 Mpc
Total Blue Magnitude M_B	15.0	8.4
Disk Scale Length h	6.8 kpc	2.1 kpc

- What is the central luminosity density of each disk (Σ_0) in $L_\odot/pc^2$?

For each galaxy fit a *maximum disk* model to its rotation curve. That is, use the formula for the rotation curve of a disk:

$$v_c^2(r) = 4\pi G \Sigma_0 h y^2 \left[I_o(y)K_0(y) - I_1(y)K_1(y)\right]$$

where $y = r/2h$ to fit a model rotation curve to the data. Important points:

- The luminosity density you calculated to begin with is not the same Σ_0 that is in the equation for v_c. The v_c equation uses the *mass surface density*, so you'll need to convert your luminosity density to a mass density using a *mass-to-light* ratio Υ^*. So treat Υ^* as a *free parameter* and vary it until the model rotation curve meets the real rotation curve at the innermost point, *but never rises above.*

- What is the value for Υ^* for each galaxy? Are these values reasonable? (*Hint: See discussions of mass-to-light ratios in Chapter 1 of BT.*)

Obviously you can't fit the entire rotation curve this way – that's the point of the next part.

- At each radius, figure out how much mass you would need to add to the galaxy to boost your fitted rotation curve up to the observed rotation curve. This mass will be due to the dark halo. Plot $M(r)$ for your halo.

- Using $dM(r) = 4\pi r^2 \rho(r) dr$, make another plot of $\rho(r)$.

- Fit a model of $\rho(r) \sim r^{-\alpha}$ in the outer regions of the galaxy. What is α?

- Comment on what you think about the physical properties of your disk/halo model for these galaxies.

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4. On the web page you will find an extremely simple FORTRAN program to calculate orbits in the logarithmic potential.

- Look at it and make sure you understand how it works.

- Use it, or make one of your own, to calculate orbits in the flattened logarithmic potential with $q = 0.9$. Do two pairs of orbits, where in each pair, the two orbits have the same initial E and L_z, but different initial phase space coordinates.

- For each orbit, plot
  - an x-y plot of the orbit,
  - an x-z plot of the orbit,
  - the orbit on the meridional plane,
  - the potential contour on the meridional plane that corresponds to the total energy of the orbit,

– $E, |\vec{L}|, L_x, L_y$, and L_z as a function of time. How well are each of these items conserved? Explain why these items are conserved as well or as poorly as they are.

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5. Use the stellar dynamics applet to study the types of orbits found in the logarithmic potential.

- Start with the singular logarithmic potential. Use an axisymmetric potential and characterize the kinds of orbits you find (loop orbits? box orbits? boxlets?). Then use progressively more flattened potentials. How do the orbits change? For a given shape, look at the orbit structure at different energies. How do the orbits change as you change energy?

- Now look at the logarithmic potential with a core. Again characterize the orbit content of the model as you go from a round to a flattened potential. Then characterize how the orbit content changesl as you change energy.

- Finally, look at the logarithmic potential with a core and an axis ratio 0.7. What is the orbit content of this model. Now turn on the black hole. How do the orbits change? Why do the orbits change? What does this mean for the evolution of such a galaxy with a black hole?