Astr327 HW #3

due March 23, 2000

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1. A simplified Oort limit. I: Theory

From the 1st moment of the cBe, we had that $\frac{1}{\nu}\frac{d}{dz}(\nu\sigma_z^2) = -\frac{d\Phi}{dz}$. Assume that the stellar disk can be treated locally as if it were a plane parallel sheet whose properties vary only as a function of z distance from the midplane. If we define a scale height for the galaxy z_0 such that $\frac{d}{dz}\ln(\nu\sigma_z^2) = -\frac{1}{z_0}$, use Gauss's Theorem to show that $\sigma_z^2 = 2\pi G\Sigma z_0$.

2. A simplified Oort Limit. II: Data

Using the Flynn and Freeman 1993 (A& A, 97, 835) catalog of K giants at the south galactic pole, do the following:

a. Calculate z, the height above the disk plane, for all the stars. Make sure to correct for the z coordinate of the sun, which is $z_\odot \sim$30pc.

b. Calculate the z component of the solar motion, and correct the velocities for the solar motion.

c. Plot v_z vs z for the stars. Now plot $\sigma_z =< v_z^2 >^{\frac{1}{2}}$ for samples of different z. How and why does it change?

d. Use the data, along with the expression you derived in Part 1, and an assumed z_0=165 pc, to calculate the local surface density of the galactic disk. *(Note: don't do the differential calculation described in class).*

e. Compare your answer to the value given in class. Describe sources of error and how they may have affected your result.

3. Globular Cluster Dynamics

Here is kinematic data for the globular cluster M4, from Peterson etal 1995 (ApJ, 443, 124). We have line-of-sight radial velocity data and proper motion data.

a. From the data, does this cluster rotate? Why or why not? Justify your answer.

b. What is the radial velocity dispersion of the cluster?

c. Using the line-of-sight radial velocity data and proper motion data together, find the distance to the cluster. Explain how you did this, and what assumptions you made.

d. Use the virial theorem to get a total mass of the cluster. Explain how you did this, and what assumptions you made.

e. Let's say, by some magic, the cluster was really 50% further than you thought it was. What is β, the anisotropy in the velocity dispersion?

4. Using the Distribution Function

Work with your partner on this one.

Okay, some of you look dubious when I told you that if you sample the DF of an object based on, say, energy alone, you got a complete model of the system. This exercise should teach you not to doubt me.

The Hernquist model has a known distribution function (it's in the paper I gave you); since it is spherical and isotropic, the distribution funtion depends on energy alone. In earlier class notes, I outlined how to make initial conditions for a particle model based on the distribution function. Here we will do this a bit more completely. Remember what we want to do: draw particles with random position (x, y, z) and velocity (v_x, v_y, v_z), draw another random variable (say ξ), then reject the particle if $\xi > f(E)$.

The problem with doing it this way is that it is very inefficient for a centrally concentrated model like the Hernquist model – you end up selecting most of your test points at large r, then rejecting them. But we can shortcut this a bit by drawing random r and v, then rejecting based on $\xi > (4\pi r^2)(4\pi v^2)f(E)$. Why is this? In a spherical, isotropic model this is simply selecting on one coordinate and integrating over the other two. The integral over the other two is simply $4\pi r^2$ for the spatial coords, and $4\pi v^2$ for the velocity coords. The only other trick is to realize that since ξ will be drawn between 0 and 1, the function we are comparing it to better be in the range 0 to 1, so we need to normalize it. So we compare to $(4\pi r^2)(4\pi v^2)f(E)/C$, where for the Hernquist model $C = 190$ (this is simply the max value of$(4\pi r^2)(4\pi v^2)f(E)$.

So we pick random r, random v, calculate E from the Hernquist model ($E = \frac{1}{2}v^2 + \Phi(r)$), then reject the point if $\xi > (4\pi r^2)(4\pi v^2)f(E)/C$

The last thing to do is to make x, y, z from r and v_x, v_y, v_z from v. To do this you need to pick randomly on a uniform sphere of θ and ϕ. To do this, pick a random $\phi(0 : 2\pi)$ and a random $\cos(\theta)$ (-1:1) (*not* a random θ – this would give you lots of points at the pole) and then apply these to the r and v of the model. Do it seperately for r and v; otherwise position and velocity will be correlated (which is bad). Here's a code snippet in fortran that does this – you can copy it into your favorite language.

Got that? It takes more time to explain it than to do it! So,

a. Using this technique, generate a 10,000 particle Hernquist model with $M = 1, a = 1$ using the distribution function. Populate it out to $r = 20$.

b. Show x-y and x-z plots of the model. Does it look right?

c. Calculate $M(r)$ from the particle distribution, and compare to the analytic result.

d. Calculate projected velocity dispersion as a function of radius from the particle distribution and compare to the analytic result.

e. Do you believe me now?

5. The Collisionless Boltzmann Equation

Describe *in words* what the cBe means, and why we compare stellar dynamics to fluid dynamics. You can write down equations – and should write down the relevant ones – but you need to describe in words what the equations and the terms within them mean.